S. ADM. BRARY

JUN 28 1929

VOLUME XXIV NEW SERIES, NO. 166

JOURNAL
OF THE
AMERICAN STATISTICAL ASSOCIATION

JUNE, 1929

CONTENTS

SPECIAL PURPOSE INDEXES FOR THE EQUATION OF EXCHANGE FOR THE UNITED STATES, 1919–1927. By Morris A. Copeland.. 109

ON THE ANNUAL REVISION OF FORECASTING FORMULAS BASED ON PARTIAL REGRESSION EQUATIONS. By A. J. Page..... 123

A NEW INDEX OF THE PHYSICAL VOLUME OF CANADIAN BUSINESS. By F. Gardiner Perry and A. G. Silverman..... 127

THE USE OF ISORROPIC LINES IN DETERMINING REGRESSION SURFACES. By Frederick V. Waugh..... 144

STATISTICS AND ECONOMIC FORECASTING. By E. M. Burns..... 152

NOTES:

Doolittle Versus the Kelley-Salisbury Iteration Method for Computing Multiple Regression Coefficients By Truman L. Kelley and Quinn McNemar..... 164

Formulas for Probable Errors of Coefficients of Correlation. By Walter Crosby Eells..... 170

A Statistical Analysis of Juvenile Employment in St. Louis. By Ralph and Mildred Fletcher..... 174

Central Collection of Social Statistics (178); Economic Outlook for 1929 (182); Fellowships of the Social Science Research Council (187); Progress of Work in the Census Bureau (189); Miscellaneous Notes (192); Members Added (203).

REVIEWS..... 207

RECENT LITERATURE..... 231

PUBLISHED QUARTERLY BY THE
AMERICAN STATISTICAL ASSOCIATION
PUBLICATION OFFICE: Rumford Press, Concord, N. H.
EDITORIAL OFFICE: Columbia University, New York City

Price $1.50 per copy $5.00 per annum

THE EMPLOYMENT CLEARING HOUSE

OF THE

AMERICAN STATISTICAL ASSOCIATION

will aid you with your employment problem. It makes no charge to employers for its services. Any member of the Association may register for a position. There is no fee.

The following positions were open in May, 1929:

Field	Pay									
	Under $1800		$1800–2400		$2500–3600		$3700–4900		$5000 and over	
	M	F	M	F	M	F	M	F	M	F
Agricultural Economics			1		1					
Business-Mfg. and Selling			1							
Drafting		1	1							
Economic and Industrial Research			2	1			1		1	
Governmental					1					
Social Research				1						
Statistical Secretarial		1								
Teaching			1		2		1			
Vital Statistics		1		1						

Members were looking for openings as listed:

Field	Pay									
	Under $1800		$1800–2400		$2500–3600		$3700–4900		$5000 and over	
	M	F	M	F	M	F	M	F	M	F
Agricultural Economics					4		2		1	
Business-Mfg. and Selling	4	1	1		3	3	4		5	
Economic and Industrial Research	4	2	2	3	2	3	1		5	
Drafting			1							
Finance	1		2		3		2		3	
Insurance		2					1			
Marketing					1		1		1	
Mathematics	1			1						
Social Research				4				1		
Teaching			1	1	2		5	2	3	
Transportation					1		1			
Vital Statistics						3	1			

COMMUNICATE YOUR NEEDS TO

MRS. ROSE L. WOODBURY

Secretary's Assistant, American Statistical Association

Room 530, Commerce Bldg., 236 Wooster St., New York City

[NOT A PAID ADVERTISEMENT]

NEW SERIES, NO. 166 (VOL. XXIV) JUNE, 1929

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

CONTENTS

SPECIAL PURPOSE INDEXES FOR THE EQUATION OF EXCHANGE FOR THE UNITED STATES, 1919-1927. By MORRIS A. COPELAND.. 109

ON THE ANNUAL REVISION OF FORECASTING FORMULAS BASED ON PARTIAL REGRESSION EQUATIONS. By A. J. PAGE 123

A NEW INDEX OF THE PHYSICAL VOLUME OF CANADIAN BUSINESS. By F. GARDINER PERRY AND A. G. SILVERMAN..... 127

THE USE OF ISORROPIC LINES IN DETERMINING REGRESSION SURFACES. By FREDERICK V. WAUGH..... 144

STATISTICS AND ECONOMIC FORECASTING. By E. M. BURNS..... 152

NOTES:

DOOLITTLE VERSUS THE KELLEY-SALISBURY ITERATION METHOD FOR COMPUTING MULTIPLE REGRESSION COEFFICIENTS. By TRUMAN L. KELLEY AND QUINN MCNEMAR..... 164

FORMULAS FOR PROBABLE ERRORS OF COEFFICIENTS OF CORRELATION. By WALTER CROSBY EELLS..... 170

A STATISTICAL ANALYSIS OF JUVENILE EMPLOYMENT IN ST. LOUIS. By RALPH AND MILDRED FLETCHER..... 174

CENTRAL COLLECTION OF SOCIAL STATISTICS (178); ECONOMIC OUTLOOK FOR 1929 (182); FELLOWSHIPS OF THE SOCIAL SCIENCE RESEARCH COUNCIL (187); PROGRESS OF WORK IN THE CENSUS BUREAU (189); MISCELLANEOUS NOTES (192); MEMBERS ADDED (203).

REVIEWS: Schultz: *Statistical Laws of Demand and Supply*, by P. G. Wright (207); Baur: *Korrelationsrechnung*, by M. Ezekiel (215); Lawrence: *Stabilization of Prices*, by R. G. Hawtrey (216); Persons: *The Construction of Index Numbers*, by W. I. King (220); Aftalion: *Cours de Statistique*, by H. L. Rietz (221); Benham: *The Prosperity of Australia*, by Z. C. Dickinson (221); Snyder: *Business Cycles and Business Measurements*, by W. A. Berridge (224); Young: *An Analysis of Bank Statistics for the United States*, by C. E. Parry (225); *Stock Dividends*, by C. O. Hardy (227); Thom: *Petroleum and Coal, the Keys to the Future*, by F. G. Tryon (229).

RECENT LITERATURE..... 231

Published Quarterly by the American Statistical Association

RUMFORD PRESS BUILDING, CONCORD, N. H.

Editorial Office, Columbia University, New York City

Entered at the post-office, Concord, N. H., as second-class mail matter

Acceptance for mailing at the special rate of postage provided for in Section No. 1103, Act of October 3, 1917, authorized April 29, 1922

Officers of the American Statistical Association

ORGANIZED NOVEMBER 27, 1839

PRESIDENT — Edwin B. Wilson

VICE-PRESIDENTS — Francis Walker, Ralph G. Hurlin, Mordecai Ezekiel

COUNSELLORS — F. Leslie Hayford, R. H. Coats, Robert E. Chaddock

SECRETARY-TREASURER — Willford I. King
Room 530, Commerce Building, New York University
236 Wooster Street, New York City

CHAPTER AND DISTRICT SECRETARY

AUSTIN (TEXAS) CHAPTER
Carroll D. Simmons, *University of Texas, Austin, Texas*

CLEVELAND CHAPTER
Harmen B. Flinkers, *Federal Reserve Bank of Cleveland, Ohio*

COLUMBUS (OHIO) CHAPTER
Harry S. Will, *66 Fallis Road, Columbus, Ohio*

LOS ANGELES CHAPTER
Ira N. Frisbee, *331½ North Beverly Drive, Beverly Hills, Calif.*

SAN FRANCISCO CHAPTER
Oliver P. Wheeler, *Federal Reserve Bank, San Francisco, Calif.*

AREA REPRESENTED AND DISTRICT SECRETARY

BOSTON AND VICINITY
Roswell F. Phelps, *Massachusetts Department of Labor and Industries, Boston, Mass.*

CHICAGO AND VICINITY
Robert B. King, *Illinois Bell Telephone Co., 212 Washington Street, Chicago, Ill.*

DENVER AND VICINITY
F. L. Carmichael, *University of Denver, Denver, Colo.*

DETROIT AND VICINITY
A. Heath Onthank, *George Harrison Phelps, Inc., Detroit, Mich.*

PITTSBURGH AND VICINITY
John N. Cover, *University of Pittsburgh, Pittsburgh, Pa.*

WASHINGTON AND VICINITY
John Rhodes, *Federal Reserve Board, Washington, D. C.*

EDITOR — Frank Alexander Ross
405 Fayerweather Hall,
Columbia University, New York City

ASSOCIATE EDITORS — Edmund E. Day, Wesley C. Mitchell, William F. Ogburn, Walter F. Willcox, Leo Wolman
30 Fifth Avenue, New York City

NEW SERIES, NO. 166 (VOL. XXIV) JUNE, 1929

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

SPECIAL PURPOSE INDEXES FOR THE EQUATION OF EXCHANGE FOR THE UNITED STATES, 1919–1927

By Morris A. Copeland, *Cornell University*

Attempts to test statistically an hypothesis regarding the casual relations between the quantity of currency in circulation and changes in the general level of prices necessarily presuppose a fairly accurate estimate of the principal items in the equation of exchange by months. Yearly figures offer a partial validation of Fisher's general analysis, but they do not enable us to investigate the lags among the items in his equation which are of chief significance for disclosing their casual relations. Figures for deposit currency and "the price level" alone do not afford an edaquate basis for such statistical analysis, as Working appears to have argued.[1] It is quite possible, for example for P, "the price level," to lag behind M', deposit currency, and at the same time for $M'V'$, check payments, to lag behind PT, the dollar volume of transactions. If this should turn out to be the case, the casual significance of a lead of M' over P is certainly considerably different from what it would be if $M'V'$, check payments, and PT should turn out to be synchronous, or if $M'V'$ should lead PT.

The recent development of statistical information in this country has made possible the construction of monthly indexes for "the price level" and "the physical volume of trade" for a period which is sufficient to yield results of some significance. These materials have been organized by Snyder to give an estimate of the changes in "the general level of prices" by months since 1913,[2] and of "the physical volume of trade" by months since 1919.[3] Snyder's indexes, combined with the

[1] Holbrook Working, *Quarterly Journal of Economics*, Vol. 37, pp. 229–257; *Review of Economic Statistics*, Vol. 7, pp. 120–133.

[2] Carl Snyder, "The Measure of the General Price Level," *Review of Economic Statistics*, February, 1928. (The original index goes back to 1875.)

[3] Carl Snyder, "The Index of the Volume of Trade: Second Revision," this Journal, June, 1928, Vol. XXIII, p. 154 *et seq.*

available data on bank debits and net demand deposits of reporting member banks and of cash in circulation, have supplied us with monthly estimates for the principal items in Fisher's equation since 1919, which place that analysis on a definite factual basis and have thrown valuable light on the cyclical relationships of the several items.

But when one attempts to use Snyder's indexes for a detailed study of lags and leads in the equation of exchange, a number of difficulties appear. Snyder's indexes are general purpose indexes, and it is an old story that general purpose indexes fit any particular problem badly. As a measure of changes in the physical volume of trade with the business cycle Snyder's trade index has many advantages over other indexes of general business, but the very characteristics which make it peculiarly adapted to this purpose are in some respects drawbacks for the purpose of estimating T as an item in the equation of exchange. Snyder's index is a weighted average of relatives to trend with all the prominent seasonal variations eliminated. It thus shows only cyclical and sporadic fluctuations of T. But for a study of the time lags among the items in the equation of exchange, it is highly important to know the seasonal variations. Again, the elimination of trends from the constituent series facilitates the use of a series like chain grocery sales as an index of the retail grocery trade (since the two may be presumed to differ chiefly in trend). But some measurement of the trend of T, made independently of any consideration of the trend of bank debits, is needed for comparing PT with $MV+M'V'$ if we are adequately to check the accuracy of the P and T indexes. Furthermore, while the index of trade, as a measure of the business cycle, is probably improved by the inclusion of deflated bank debits, it is clear that such a series should not be included in an estimate of T as an item in the equation of exchange for two reasons: (1) Bank debits are a fairly accurate measure of $M'V'$. They are clearly not one of the special groups of "pt's" that go to make up PT (or $\Sigma\, pt$). (2) In order to include debits in T it is necessary, because they represent no special group, to deflate them with a general index of prices. Thus an important element in Snyder's T is not determined from empirical data independently of the other items in the equation of exchange but as $\frac{M'V'}{P}$. This element has a weight of 12 per cent. New capital flotations, as a constituent T series with a weight of 2 per cent in Snyder's index, are also subject to the objection that they have been deflated by P. Both Snyder's trade index and his index of the general price level are, for purposes of their specific use in the equation of exchange, open to certain criticisms of weighting. And

finally, indexes of P and T alone, for reasons that will presently be noted, give an incomplete picture of the so-called "goods side" of the equation.[1]

Certainly none of these points is intended as a criticism of Snyder's indexes as general purpose indexes. But it has seemed desirable, in view of the difficulties in his indexes for the specific purpose in hand, to construct a new set of special purpose indexes upon which a study of the lags and leads in the equation of exchange can be based. The present paper aims to set forth these new indexes, including certain corrected indexes for the so-called money side of the equation. In a subsequent paper some of the leads and lags will be analyzed.

It has often been said that the equation of exchange is a tautology. If the equation is stated, "the quantity of cash disbursed in any period in discharge of financial obligations = the value of financial obligations discharged in that period," it may be said to be a tautology. But when one substitutes Σpt for "value of financial obligations discharged," where "t" means number of units of a good purchased during the period and "p" the average market price charged for the good, the second part of the equation refers more nearly to "obligations incurred to make money payments" than to "obligations discharged." In this form the two sides of the equation are separate, empirically determinable magnitudes, and it even becomes a pertinent question whether the equation is precisely true.

Let us consider first the problems of measurement involved in the "obligations discharged" side of the equation. It has been customary to represent this as $MV+M'V'$, where MV refers to cash payments and $M'V'$ refers to payments by check. The chief source of information on this side of the equation is "debits to individual accounts" in 141 centers. These include approximately 80 per cent of all such debits[2] in the United States. A debit to an individual account may arise from (1) a payment by check; (2) the withdrawal of cash from a bank (either from a time or a demand account); (3) a withdrawal from a time account for immediate transfer to a demand account. Fisher has estimated that a dollar withdrawn from a bank makes on the average less than two payments before it is redeposited. Apparently, then, the debits data include a large proportion of cash payments (slightly in advance of their actual occurrence) as well as payments by check. They also include under item (3) certain debits which do not properly fall either under MV or $M'V'$—debits which are merely changes from one form of deposit to another. To include them and then to include

[1] These points are developed more fully below.

[2] See the writer's estimate, this JOURNAL, September, 1928, p. 303.

also the checks drawn on the demand deposits so created would be to count these cash disbursements twice.

Because our present information does not enable us to separate accurately on a monthly basis what Fisher called MV from what he called $M'V'$, we will use only the one set of symbols MV for the left-hand member of the equation, as representing all cash disbursements. But we may correct the monthly debits series by more accurate annual estimates to meet the difficulties that have just been noted.

Annual estimates for this purpose have been made as follows:

1. From annual estimates of total United States debits [1] two times the estimated total time deposits of all United States banks (as of approximately June 30) was subtracted, *i. e.*, it was assumed that time deposits turn over twice a year.[2] The result = estimated debits to demand accounts.

2. No. 1 was divided by demand deposits of all banks to determine their approximate rate of turnover. It was assumed that the annual turnover of money in circulation was the same as this slightly exaggerated demand deposit turnover,[3] and cash payments estimated by multiplying V by money in circulation. One half of the result was added to the estimated annual debits to demand accounts (on the assumption that about half of cash payments is already included in No. 1). The result agrees approximately with an independent estimate for 1919 and 1921 based on Fisher's 1909 figure for cash turnover.

3. Debits to Federal Government accounts in Federal Reserve Banks were added to No. 2 to give estimates of total cash disbursements. The monthly data on debits in 141 centers were adjusted to agree with these annual estimates.

The result is perhaps imperfectly representative of some types of payments, particularly rural payments, but on the whole is probably a fairly reliable index of MV.

M is also an empirically determinable quantity. Much the largest element in M is demand deposits. Information on demand deposits of banks which are members of the Federal Reserve System is available for dates of call—not monthly. Figures for dates of call have therefore been interpolated on the basis of net demand deposits [4] to get monthly

[1] See the present writer, *loc. cit.*

[2] *Cf.* W. Randolph Burgess, *Federal Reserve Bulletin*, Vol. 9, p. 563. A questionnaire submitted by the writer answered by some twenty-five banks out of seventy-five, showed wide variations about this figure. In the absence of more information it may be accepted as the best present guess.

[3] Slightly exaggerated because cash withdrawals are not excluded.

[4] Prior to April, 1923, these monthly data are for reporting member banks; thereafter for all member banks. The call-date items represent individual "demand deposits" less "exchanges for clearing house," "items with Federal Reserve banks in process of collection" and "outside checks."

figures. Net demand deposits include certain net items "due to other banks" and are unsatisfactory as an index of deposit currency, but they are the best available basis for interpolation. To these monthly estimates of deposit currency made by interpolation, one half[1] of the revised figures on money in circulation has been added to make the index of M comparable with that of MV, which includes both check payments and a part of payments with cash. That is, M = demand deposits of member banks, interpolated on monthly figures for net demand deposits + one half of the currency in circulation. The resulting index of M represents the average for each month (in the case of deposits it is an average of figures for first and last of the month). The annual averages of this index were corrected to agree with estimates of total demand deposits plus total currency in circulation, in a manner analogous to the corrections of the annual averages of the MV index. One theoretical qualification should perhaps be noted. To a considerable and probably an increasing extent the line between deposit currency and savings deposits is blurred. Savings deposits are used today, to an extent which it is impossible to determine, as a cash reserve, where formerly this reserve may have been in the form of a demand deposit. If we cannot say with certainty precisely what is meant by "currency in circulation," we shall do well to remember this difficulty in attributing casual primacy to M.

We may consider next the problem of measurement of the dollar volume of obligations incurred to make money payments. Theoretically these data may be compiled on either of two bases: (1) as obligations of business enterprises or other parties, incurred to make money payments, and (2) as claims to receive money payments acquired by the prospective recipients. Actually most of our figures are from the point of view of the prospective payee, representing either cash receipts or additions to his "receivables." If we had complete figures on this basis it would be desirable, in order to avoid double counting, not to include any figures for cash disbursements or additions to "payables." In estimating PT for a single country like the United States, therefore, we should include merchandise and invisible imports as representing sales of foreign exchange, and so receipts of the foreign exchange bankers, but theoretically we should not include exports as a separate item, since these would all appear as sales of commodities, securities, services, etc.[2] As a matter of fact, figures on trade are so

[1] Since cash is a small item compared to deposits and agrees approximately in its movements, the coefficient "one half" gives about the same results as would zero or one.

[2] This seems a more logical view than one half (exports + imports), the one Fisher states in *Purchasing Power of Money*, 2d ed., p. 483, especially since he clearly distinguishes the two bases of compilation on p. 373.

incomplete as to make it desirable to include merchandise exports in the monthly indexes, and probably also government disbursements.

In order to determine the importance of the several available monthly series for purposes of assigning weights, and in order to appraise the reliability of an index based on data now available, it is desirable to know approximately the annual dollar volume of money payments to which each main class of transactions gives rise. Crude estimates of this sort have been made for 1919 and 1921.[1] These estimates bring out several important points:—

1. Financial transactions are of outstanding importance. The following groups of transactions probably amounted to two hundred billion dollars, or over 30 per cent of the total of cash disbursements in 1919 (about six hundred billion dollars)—short-time loans, resales and repayments, capital flotations and retirements, stock exchange and over-the-counter security sales, commodity exchange transactions, time deposits, foreign exchange sold, interest and dividend payments.

2. Not all obligations to make money payments arise from the sale of a specifiable physical volume of goods or services at a specifiable price. Most taxes and charity subscriptions fit this formula badly. The flotation of a new stock issue involves a sale, but practically it seems hardly feasible to find two time series, one to represent prices and one physical volume of sales of new securities. Again, the making of a loan and the payment of interest and principal are not easily to be treated as a "p" $\times$ a "t." And there are a number of other transactions that are at least equally difficult to treat as a "p" $\times$ a "t." The following items probably amounted to at least two hundred billion dollars in 1919, or over 30 per cent of the total MV—short-time loans, resales and repayments, payments from one checking account to another held by the same depositor, capital flotations and retirements, time deposits made, taxes, insurance premiums and benefits, interest and dividend payments, reimbursements of agents for expenses incurred, cash gifts, receipts of churches and clubs. In view of the importance of these transactions it seems necessary to include them in our indexes for the right-hand side of the equation. We shall designate this general group of transactions R, and write the equation $MV = k(PT+R)$.[2] And our problem is enlarged to include the construction of an index of $PT + R$.

3. The information available on a monthly basis regarding $PT+R$

[1] For certain groups not represented in the indexes these estimates are not yet completed. The methods employed in the several estimates are somewhat complicated and cannot be set forth here. It is hoped that a description of them will be available shortly.

[2] k is an arbitrary constant. It enables us to express the right-hand member in index number form on any desired base.

is far from complete. It probably represents less than one third of the total $PT+R$. Data on R are particularly scarce; it seems improbable that as much as one sixth of R is represented by current monthly figures. Some of the more important items for which information is largely lacking are, in approximate order of magnitude, short-time loan transactions (especially brokers' day loans), over-the-counter bond sales, payments from one checking account to another held by the same party, activity of time deposits, payrolls (other than railroad and manufacturing), personal and professional service enterprise revenues, rents, real estate sales, local government revenues.

The indexes which are available for the various groups of transactions may be in the form of "p's," "t's," or pt's," a price series, a physical volume series, or a dollar volume series. Any two of these three, referring to the same set of transactions, will suffice. But in a number of cases either only one is available or if two are available, they refer to two partly overlapping but not quite identical groups of transactions. A "t" for real estate sales is available (for 41 cities), but no satisfactory corresponding "p" or "pt." For house rents we have a "p" but no corresponding monthly "pt" or "t." In the case of manufacturing revenues we have a "t" (output) and can adapt existing "p" indexes to represent a manufacturing "p." Moreover, we can correct the resulting "pt's" biennially by the census figures. Wherever a "p," a "t," and a "pt" series were available such a correction has been made in the series employed in the P, T, and $PT+R$ indexes. And more accurate annual data have, so far as possible, been used to correct the monthly figures. The constituent series employed, then, consist of corresponding "p's," "t's," and "pt's," such that for each set "p" $\times$ "t" = "pt," and such that the annual averages of the monthly "p's," "t's," or "pt's" agree with corresponding annual data.

In constructing the indexes an approximation to Fisher's ideal formula has been used. This means that the basic index is the $PT+R$ index. This is a weighted average of relatives to 1919, with the weights based on the 1919 dollar volume estimates. Since the monthly data have been corrected by annual figures, this index is not materially affected by the choice of 1919 as a weight-base year.[1] The construc-

[1] If W represents any pt or r and the year is indicated by the subscript, we have

$$ {}^{b}(PT+R)_x = \frac{\sum W_b \frac{(W_x)}{(W_b)}}{\Sigma W_b} $$

where b indicates the base year. In other words,

$$ {}^{b}(PT+R)_x \times {}^{x}(PT+R)_y = {}^{b}(PT+R)_y \text{ and } {}^{b}(PT+R)_x = {}^{x}(PT+R)_b{}^{-1}. $$

Both the time-reversal and base-shift tests are satisfied. The factor reversal test, of course, does not apply.

tion of the P and T indexes is based on the PT constituent of the $PT+R$ index. So-called aggregative indexes of P and of T are first constructed, using the nine-year averages of the "pt's," "t's," and "p's" as basic. The resulting preliminary indexes of P and T are then adjusted to make $P \times T$ equal the PT constituent of the $PT+R$ index.[1] The dependence of the P and T indexes upon the $PT+R$ index and upon each other, both in determining the weights for the preliminary indexes and in making the corrections, raises an interesting theoretical question: It is clearly proper to speak of $PT+R$ as an independent empirical quantity, but is it proper to speak of either P or T as independent? Are they not rather interdependent?

One question which arises regarding the weights deserves a word of comment. Samples of "$p's$" are more likely to be representative than samples of "$t's$" because the effect of local conditions on prices is more likely to be evenly diffused over the country than the effect on output. Moreover, the dollar volume to which a given "p" and the corresponding "t" have reference may differ even where there is close correspondence in the definition of the sampled groups so that it is strictly proper to multiply the "p" by the "t." In assigning dollar volume figures to this "pt" the inclusiveness of the "t" has been the primary consideration. And in general the chief basis of weighting in the $PT+R$ index has been a compromise between the dollar volume included in the sample "pt" series and the dollar volume of the group which it represents.

It may be interesting to compare some of the weights employed in the $PT+R$ index with the corresponding weights in Snyder's indexes of the general price level and the volume of trade. In Snyder's trade index the weights apply to relatives to (seasonal x trend). In his price index they apply to relatives to 1913 as 100 per cent. The weights for the $PT+R$ index refer to relatives to 1919–27 average as 100 per cent. The principal items are given in Table I.

[1] According to Fisher,

$$P_2 = \sqrt{P_1 \times \frac{(PT)}{T_1}} \quad \text{and} \quad T_2 = \sqrt{T_1 \times \frac{(PT)}{P_1}}$$

where the subscript 1 indicates the preliminary P and T, and (PT) is the PT constituent of the $(PT+R)$ index. Where the correction is small we may write $P_2 = P_1 + \frac{1}{2}\left((PT) - P_1T_1\right) 1/T_1$ for

$$P_2T_1 = \sqrt{(P_1T_1)^2 + \frac{2}{2}P_1T_1\left((PT) - P_1T_1\right) + (\tfrac{1}{2} - \tfrac{1}{2})\left((PT) - P_1T_1\right)^2}.$$

To the corrected P_2 and corrected T_2 so obtained, the series on rents and number of real estate transfers respectively were added. The nine year averages of P_2 and T_2 had weights of 98 per cent and the extra series of 2 per cent.

TABLE I

COMPARISON OF WEIGHTS APPLIED TO RELATIVES IN SNYDER'S INDEXES AND $PT+R$ INDEX

	Snyder's trade relatives to "normal"	$PT+R$ relatives to 1919–1927 average	Snyder's price level relatives to 1913
New securities and corp. formed	3	4.5	0
Stocks and bonds	1	13.0	10
Brokers' loans paid off	0	1.8	0
Subtotal	4	19.3	10
Railroads	8	3.4	5
Panama Canal	1	0	0
P. O. Tel., Tel. and Elec. Power	7	2.9	0
Subtotal	16	6.3	5
Payrolls	4	5.8	15
Real estate sales and rentals	2	2.0*	15
Subtotal	6	7.8	30
Govt. disbursements	..	1.0	..
Govt. receipts	..	.7	..
Interest	..	2.3	..
Dividends	..	2.2	..
Subtotal		6.2	
Production	20	23.2	} 25
Wholesale trade	5	7.8	
Foreign trade	7	2.9	
Building constr.	5	4.3	10
Retail Trade	19	18.4	20
Subtotal	56	56.6	55

* Included only in P and T indexes, not in $PT+R$.

It will be noticed that the disagreements between Snyder's two indexes are quite as marked as those between either of them and $PT+R$. In part this may be due to differences in information available on "*p's*" and on "*t's*" in the several groups. Also, the basis of stating the price level weights may be slightly misleading. But it is clear that important discrepancies remain. In general, Snyder has given little weight to financial items (other than debits), especially in his trade index. And he has given heavy weights to such stable items as public utilities in the trade index, and wages and rents in the price level, so that we should expect his indexes to be more stable than the special purpose index numbers. In Table II comparisons of yearly averages show what might be expected under these circumstances.

When the indexes were constructed a final correction was applied to the MV, V, $PT+R$, and T indexes, so as to express them on the basis of volume or velocity per working day in each month. It is essential to make such a correction if time lags are to be investigated. Otherwise the synchronous correlation will contain a spurious element,

which will make it higher relatively to correlation with a lag either way than would be the case without the arbitrary fact that March has more working days than February. But what correction should be applied? Not only do different kinds of business work different numbers of days in the month, some every day; but also we have come, for certain purposes, to treat the arbitrarily differing months as if they were all equal. Dividend payments, salaries, most rentals, income taxes, and many other payments are made without regard to the number of days in the month. Perhaps an index number of working days should be constructed to take account of all these facts. But wanting that, it is certainly better to divide the indexes by a figure for working days which treats Sundays and certain holidays as non-working. The following holidays are allowed for as one day: January 1, May 30, July 4, Labor Day, Thanksgiving, and Christmas. (When they fall on Sunday, January 1 and May 30 are counted as half days, the other holidays as whole days.) February 22, Good Friday, and Columbus Day are counted as half days.

TABLE II

ANNUAL AVERAGES OF INDEXES OF ITEMS
IN THE GOODS SIDE OF THE EQUATION OF EXCHANGE

	1919	1920	1921	1922	1923	1924	1925	1926	1927
$PT+R$	100	103	78	88	96	97	115	119	134
PT	100	108	78	88	99	100	119	123	135
P	100	111	90	85	89	89	93	93	96
T	100	98	89	107	114	115	132	134	143
Snyder's P	100	112	94	91	95	96	98	99	99
Snyder's T	105	102	92	99	104	102	107	107	107

The finally resulting indexes and the constituent series with their weights are shown in the appendix. It will be the purpose of a subsequent article to analyze some of the relations among these indexes. It is hoped that others may find them useful for still further studies of the equation of exchange.

APPENDIX

CONSTITUENT SERIES AND ANNUAL AND BIENNIAL CORRECTION DATA: P, T, AND PT+R INDEXES

	PT+R	P	T
(1) Railroad business	Ry. Op. Rev. Class I R. R.s I. C. C.	pt+t (% changes in rate level)	Monthly and annual traffic indexes *
(2) P. O., Tel., and Tel. Gas and elec. business	Pub. Util. Rev.† Index	Pub. Util. rate index ‡	pt+p
(3) Construction	Contracts—27 and 36 states spliced at Jan. 1923—F. W. Dodge Corporation	Eng. N. Rec. Index of construction cost. Yearly index 1/10 (6 ×B. L. S. Materials cost+4×B. L. S. wage rate)	pt+p
(4) Retail trade	Writer's index Harvard Bus. Rev., Jan., 1929	B. L. S. cost of living ex., rent, fuel and light interpolation on N. I. C. Bd. Indexes ¼ (clothing+2× food+misc.)	pt+p
(5) Manufacturing	p×t—value of mfg. Biennial Census	B. L. S. Wholesale Price Indexes ½ (2×consumers' goods+producers' goods)	Dept. of Commerce production index. Thomas yearly production index
(6) Raw materials	p×t—Annual estimates of value of agricultural and mineral products based on Bur. of Ag. Ec. and Bur. of Mines data	F. R. Bd.—B. L. S. raw material price indexes	Dept. of Commerce index (mineral production and agricultural marketings)
(7) Wholesale trade	F. R. Bd. Rev. Index	B. L. S. wholesale prices	pt+p
(8) Factory payroll	F.R. Bd. Payroll Index ×No. of working days, Biennial Census	pt+t	F. R. Bd. employment × No. of working days, Biennial Census
(9) Railroad payroll	Class I R. R.s I. C. C. (1919 and 1920 estimated monthly figures)	Av. hourly compensation Class I R. R.s (1919 and 1920 monthly estimates)	pt+p
(10) N. Y. Stock Exchange—stock sales	p×t	Stock price index of Standard Stat. Corp.	No. of shares (Annalist)
(11) N. Y. Stock Exchange—bond sales	p×t	N. Y. Times price index	Par value of bonds sold—Dow Jones
(12) Mdse. exports	Dept. of Commerce	F. R. Bd. index and writer's index in 1926–27 Dept. of Commerce annual index	pt+p
(13) Mdse. imports	Dept. of Commerce	F. R. Bd. index and writer's index in 1926–27 Dept. of Commerce annual index	pt+p
(14) Commodity futures	Estimates based on tax receipts—Internal Rev. Bur.	Writer's index based on cotton, corn, wheat and oats spot prices	pt+p
(15) Fed. Govt. disbursements	Ordinary and special disb. minus interest on public debt		

APPENDIX—*Continued*

	PT+R	P	T
(16) New capital flotations	Index based on municipal bonds (Bond Buyer); public debt transactions; foreign issues (Dept. of Comm.); and new corporate securities (J. of C and C. and F. C. spliced at Jan. 1924)§		
(17) Federal Gov't receipts	Ordinary receipts		
(18) Brokers' loans paid off	Through stock clearing corporation (trend corrected by loans outstanding—F. R. Bd.)		
(19) Interest payments	J. of C.+Fed. Debt. interest		
(20) Dividend payments	J. of C.		
(21) Life insurance premium collections	Association Life Insurance Pres.		
(22) Real estate rentals		B. L. S. Rent Interpolated on N. I. C. Bd. shelter index	
(23) Real estate sales			Number of real estate transfers in 41 cities

* Monthly index: ton-miles, weighted 4; carloadings, weighted 4; passenger-miles, weighted 2. Annual index substitutes tons of revenue freight for carloadings.

† P. O. receipts at fifty cities, weighted 2; telephone revenues (12 companies reduced from 13 by consolidation—Department of Commerce), weighted 2; telegraph and cable operating revenues (Western Union and Postal), weighted 1; gas and electric company gross earnings (95 companies and 78 companies spliced at 1927, 78 and 73 companies spliced at 1925, Department of Commerce), weighted 5; monthly figures in 1919 for first four series were estimated on seasonal indexes and annual data.

‡ B. L. S. gas prices (linear interpolation for monthly figures), weighted 2; annual telephone revenues per Bell station at end of year and electric revenues per kilowatt-hour (interpolated on a linear basis for monthly figures), weighted 2 and 3 respectively; postal rates and telegraph rates indexes made on the basis of announced percentage changes, weighted 2 and 1 respectively.

§ Permanent municipal bonds and public debt receipts or disbursements (the larger of these two figures for each month was used) were each weighted ½. The other series each had a weight of 1 on a dollar volume basis.

WEIGHTS USED IN PT+R INDEX (1919=100%); RATIO OF ESTIMATED 1919 DOLLAR VOLUME OF EACH CONSTITUENT SERIES TO TOTAL FOR ALL SERIES IN THE INDEX IN PER CENT; AND RATIO OF ESTIMATED TOTAL 1919 DOLLAR VOLUME OF EACH CLASS OF TRANSACTION REPRESENTED TO TOTAL FOR ALL CLASSES REPRESENTED, IN PER CENT

	PT+R weights	Dollar volume percentage ratios	
		Total of each group represented	Sample series
R. R. revenues	3	2	4
Pub. Util. rev.	2	2	1
Constr. contracts	3	2	2
Retail trade	16	14	3
Manufactures	20	22	16
Raw materials	6	6	12
Wholesale trade	9	14	3
Factory payroll	5	5	2
R. R. payroll	1	1	2
Settlements for stock sales, N. Y. Exchange	9	7	13
Settlements for bond sales, N. Y. Exchange	2	1	2
Merchandise exports	2.5	3	6
Merchandise imports	1.5	1	3
Commodity futures	1	1	3
Fed. Gov't disbursements	3	4	8
Capital flotations	7	9	11
Fed. Gov't receipts	1	2	5
Brokers' loans paid off	1	1*	1
Interest paid	2	1	2
Dividends paid	2	1	1
Life insurance premiums received and agents' remittances	3	1	0†
	100	100	100

* It is assumed that this is only twice the sample, which is not very representative of all brokers' loans paid off. The figure for the whole group would be much larger.
† Less than one half of one per cent.

SPECIAL PURPOSE INDEXES FOR THE EQUATION OF EXCHANGE, UNITED STATES, 1919–27

(1919 Average=100%)

INDEX OF P

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual average
1919	96	95	94	96	97	99	102	104	102	103	105	108	100
1920	111	110	111	118	115	115	116	112	111	107	103	98	111
1921	96	95	93	92	91	88	88	87	87	87	86	85	90
1922	84	84	83	82	83	83	86	85	85	87	87	88	85
1923	88	89	90	90	89	88	88	87	89	89	89	89	89
1924	89	89	89	88	88	87	87	88	88	89	90	91	89
1925	92	93	92	91	91	91	93	93	92	93	94	95	93
1926	95	94	92	92	92	93	93	93	93	92	92	93	93
1927	93	94	94	94	95	95	93	96	98	98	98	101	96

INDEX OF T PER WORKING DAY

1919	83	84	89	97	98	102	101	99	106	115	116	110	100
1920	99	102	106	104	98	92	90	92	98	100	103	97	98
1921	84	81	85	87	90	88	82	84	93	99	96	97	89
1922	87	91	100	112	109	111	100	100	112	120	120	118	107
1923	105	111	115	118	118	116	104	103	115	118	119	124	114
1924	111	111	111	114	105	105	103	107	118	122	137	135	115
1925	121	123	122	124	129	121	122	123	135	151	157	155	132
1926	129	129	137	128	122	129	127	136	141	147	138	145	134
1927	125	137	151	145	143	139	132	139	151	153	147	155	143

INDEX OF PT PER WORKING DAY

1919.....	80	80	84	92	95	100	102	103	107	117	121	118	100
1920.....	109	111	116	116	112	107	103	102	108	106	106	95	108
1921.....	80	76	78	78	80	77	71	73	80	85	82	82	78
1922.....	72	75	81	91	89	91	84	83	94	103	103	102	89
1923.....	91	97	102	104	103	100	89	88	100	103	104	108	99
1924.....	96	97	96	98	90	90	88	93	102	106	121	122	100
1925.....	110	112	115	110	115	109	111	112	123	138	137	135	119
1926.....	120	119	124	116	110	118	115	124	130	134	126	134	123
1927.....	115	127	133	134	134	130	123	131	146	148	143	154	135

INDEX OF PT+R PER WORKING DAY

1919.....	91	85	86	91	97	103	102	98	111	111	111	116	100
1920.....	106	104	114	112	103	107	98	94	104	101	100	96	103
1921.....	80	74	79	79	79	80	71	70	80	83	79	84	78
1922.....	72	74	84	90	85	93	82	79	91	100	97	104	88
1923.....	97	92	99	101	99	99	87	82	96	98	98	106	96
1924.....	95	93	96	96	87	89	86	88	100	103	113	121	97
1925.....	108	108	115	107	109	107	107	104	119	131	131	134	115
1926.....	120	115	125	114	105	116	112	116	125	128	120	133	119
1927.....	117	124	137	133	130	132	119	125	143	146	141	157	134

INDEX OF M

1919.....	96	96	96	96	98	98	99	101	103	105	107	108	100
1920.....	110	110	110	110	110	110	110	110	110	110	110	107	110
1921.....	104	103	102	100	100	100	98	97	96	97	97	96	99
1922.....	96	94	94	95	97	98	98	97	98	99	99	100	97
1923.....	100	101	100	99	100	101	100	101	101	102	102	102	101
1924.....	102	102	102	102	104	106	107	108	110	112	113	114	107
1925.....	114	114	113	112	112	113	114	114	116	117	118	118	114
1926.....	117	117	116	116	117	117	117	117	117	118	118	118	117
1927.....	119	118	120	120	121	121	121	121	122	124	124	126	121

INDEX OF V PER WORKING DAY

1919.....	95	84	87	90	98	108	108	98	101	108	112	111	100
1920.....	107	93	98	100	95	95	95	86	92	98	102	103	97
1921.....	98	82	82	83	85	85	85	75	86	91	90	99	87
1922.....	93	89	94	101	97	103	94	83	94	104	98	105	96
1923.....	104	98	102	105	100	105	94	79	91	93	97	107	98
1924.....	101	95	98	96	94	97	92	87	90	92	98	106	96
1925.....	107	98	102	98	104	104	100	90	98	108	108	112	102
1926.....	117	102	112	110	101	105	105	96	100	109	100	116	106
1927.....	116	108	113	112	111	111	109	101	115	117	114	125	113

ON THE ANNUAL REVISION OF FORECASTING FORMULAS BASED ON PARTIAL REGRESSION EQUATIONS

By A. J. Page, *Rothamsted Experimental Station, Harpenden, Herts*

The fitting of regression lines to statistical data which have been obtained by experiment or other means more or less under control, has been widely applied, and has produced valuable results, but the method is much more restricted when applied to statistics relating to economic and social problems on account of the complexity and variety of the underlying causes at work. Cases, however, do arise where the main factors are few and easily distinguishable, thereby enabling exact mathematical procedure to be followed with some degree of success. It has been found possible, for example, to work out prediction formulas for the preparation of official crop forecasts, though a difficulty often arises owing to the comparative shortness of the time series available as a basis of the calculations, which may be due not to the absence of figures for earlier years, but to changes in conditions during the course of time which would render the material of a long series obviously heterogeneous. When all such causes and conditions have reached a fairly stable state of equilibrium amongst themselves, the addition of each successive year's data, as time goes on, lengthens the series available, and enables the dependent variate to be determined with greater precision.

The procedure to be followed is simple and does not involve the use of correlation coefficients. It may be studied in full in such a work as R. A. Fisher's *Statistical Methods for Research Workers*. The arithmetical work involved may, however, be laborious, and the object of the present note is to show how, when once the necessary equations have been obtained from the data available for any given period of years, these equations may easily be brought up to date by the incorporation of the figures for a new year, without having to recalculate *ab initio* all the numerical coefficients involved.

If the expected value of a dependent variate Y is expressed in terms of a number of independent variates $x_1, x_2, x_3, \ldots\ldots\ldots, x_r$ in the form of a partial regression equation

$$Y = b_1x_1 + b_2x_2 + \ldots\ldots, + b_r x_r, \qquad (1)$$

the partial regression coefficients $b_1, b_2, \ldots\ldots, b_r$ are obtained from the simultaneous equations

$$\left.\begin{array}{l} b_1\Sigma(x^2_1) \quad +b_2\Sigma(x_1 x_2)+ \ . \ . \ . \ . +b_r\Sigma(x_1 x_r)=\Sigma(x_1 y) \\ b_1\Sigma(x_1 x_2)+b_2\Sigma(x^2_2) \quad + \ . \ . \ . \ . +b_r\Sigma(x_2 x_r)=\Sigma(x_2 y) \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ b_1\Sigma(x_1 x_r)+b_2\Sigma(x_2 x_r)+ \ . \ . \ . \ . + \ b_r\Sigma(x^2_r)=\Sigma(x_r y) \end{array}\right\} \quad (2)$$

where y, x_1, x_2, ... , x_r are all measured from their respective means. If they were not so measured and, instead, the crude values used, there would be no great trouble or labor involved in bringing the above equations (2) up to date by the addition of a new term to each of the series which forms the numerical coefficients of the b's. However, since all the means become altered, every one of the terms in the different series is affected.

Suppose the equations (2) to be based on figures for t years, and that the inclusion of the $(t+1)^{\text{th}}$ year increases the mean from which x_1 is to be measured by d_1. The new value of the variate evidently exceeds the old mean by $(t+1)d_1$ and the new mean by td_1. The sum of the squares of the deviations for the first t years from a value differing by d_1 from their mean is evidently increased by

$$td^2_1$$

while the new square is $t^2d^2_1$, making a total increase in the coefficient of b_1 in the first equation of $t(t+1)d^2_1$.

Moreover, if the absolute values of the variate for the $t+1$ years are represented by x_{11}, x_{12}, ... $x_{1\cdot t+1}$

$$d_1=\frac{1}{t+1}\sum_{p=1}^{+1}x_{1p}-\frac{1}{t}\sum_{p=1}^{t}x_{1p}$$

or

$$d_1=\frac{1}{t(t+1)}\left\{tx_{1,t+1}-\sum_{p=1}^{t}x_{1p}\right\};$$

d_1 is defined by the equation and the adjustment of the coefficient due to the inclusion of the new year's data consists in merely increasing it by

$$\frac{1}{t(t+1)}\left\{tx_{1,t+1}-\sum_{p=1}^{t}x_{1p}\right\}^2. \quad (3)$$

Similarly, the sum of the products of the deviations from the mean of the variates x_1 and x_2 will receive an adjustment

$$\frac{1}{t(t+1)}\left\{tx_{1,t+1}-\sum_{p=1}^{t}x_{1p}\right\}\left\{tx_{2,t+1}-\sum_{p=1}^{t}x_{2p}\right\}. \quad (4)$$

The system of equations (2) may then be brought up to date merely by calculating for each variate the difference between t times its new value and the sum of the previous values, and adjusting the coefficients by the addition of the terms given by (3) and (4).

As a practical example of the method, figures used for the official forecasts of the exportable surplus of rice from the province of Burma may be cited. The full set of equations for the whole province are too elaborate to be quoted here, as no fewer than six b's in six simultaneous equations are involved, but the subsidiary equations from the port of Akyab which deal exclusively with the produce of the rice lands in the Arakan division of the same province may be quoted as they involve only two parameters. For the years 1902 to 1926, omitting the years 1915 to 1920 which were affected by war conditions, the exportable surplus is given by the equation

$$Y = -27.390 + .257x + .029x'$$

where Y is the difference in hundreds of tons between the exports of any year and those of the preceding year, x is the difference between the area harvested in hundreds of acres multiplied by the condition figure of the crop for the same year, and the corresponding product of the preceding year, whilst x^1 is the similar difference between the similar products for the preceding and second preceding year. In other words, and expressed algebraically, if a_1, a_2, a_3 are the areas harvested in three consecutive years and c_1, c_2, c_3 the corresponding condition figures of the crops

$$x = a_3\,c_3 - a_2\,c_2, \qquad x' = a_2\,c_2 - a_1\,c_1.$$

Now the coefficients .257 and .029 were obtained as the approximate solutions of the equations

$$\begin{aligned} 19{,}295{,}470b_1 - 11{,}971{,}249b_2 &= 4{,}608{,}535 \\ -11{,}971{,}249b_1 + 33{,}387{,}878b_2 &= -2{,}096{,}725 \end{aligned}$$

where the coefficients are the sums of squares, or products, for 17 years.

The figures for the succeeding year 1926–27 were $x = -219$, $x' = -354$ and $y = -280$; t is 17, $t+1$ is 18 and the crude totals $\Sigma(x)$, $\Sigma(x')$ and $\Sigma(y)$ for 17 years were 6,368, 1,702, and 1,285 respectively. For the three variates x, x' and y we have therefore

$$\begin{aligned} 17\times(-219) - 6{,}368 &= -10{,}091 \\ 17\times(-354) - 1{,}702 &= -7{,}720 \\ 17\times(-280) - 1{,}285 &= -6{,}045 \end{aligned}$$

of which the squares and products divided by $17\times18(=306)$ gives the adjustments to the coefficients:

332,772	254,583	199,347
254,583	194,766	152,508

and on adding these to the coefficients of the old equations we arrive at the following new equations

$$\begin{aligned} 19{,}628{,}242b_1 - 11{,}716{,}666b_2 &= \quad 4{,}807{,}882 \\ -11{,}716{,}666b_1 + 33{,}582{,}644b_2 &= -1{,}944{,}217, \end{aligned}$$

giving a new regression equation

$$Y = -37.551 + .266x + .035x'.$$

The benefit of this simple labor-saving formula is not, however, seen in its fullest extent in the example given since considerations of space prevented heavier work being displayed.

A further advantage of the method is that in some cases, as in the one given above, where the x's are known early in the year but the y's are not known until the year has closed, the equations can be revised, as far as all the coefficients on the left-hand sides of the equations are concerned, and kept in readiness for the insertion at the earliest possible moment of the corrections to the $\Sigma(xy)$ items, when the new solutions for the b parameters can quickly be found. Indeed, using the left-hand sides of the equations only, a matrix of multipliers may be prepared, which, with the new values for the right-hand sides, will supply the new solutions by direct multiplication and addition, thus anticipating nearly all the labor, not only of preparing, but of solving the equations.

A NEW INDEX OF THE PHYSICAL VOLUME OF CANADIAN BUSINESS[1]

By F. Gardiner Perry, *Babson Statistical Organization*, and A. G. Silverman, *Massachusetts Institute of Technology*

The calculation of the index numbers presented below, first projected several years ago, was started in the summer of 1927. In order to furnish a physical volume basis for the study of the interrelations of Canadian industries and major groups and for comparison with business fluctuations in the United States, it was desired to cover on a monthly basis as long a period as possible. This gave rise to numerous problems in method as well as in the collection and choice of series. The selection of weights for the subjects in a rapidly developing economic situation such as that in Canada at the outset forced the view, not often enough explicitly recognized, that the making of index numbers, for even binary comparisons, is at best an attempt to do as logically as possible the illogical. A compromise formula was the result. It was also early concluded that for many series adjustment for seasonal variation determined on a constant basis is not only inadequate but sometimes worse than no adjustment at all. Moving seasonal indexes were used to adjust practically all series. It is hoped that the calculation of the indexes presented below will help to accumulate evidence for the use of moving measures to characterize the changing central tendencies of economic variables.

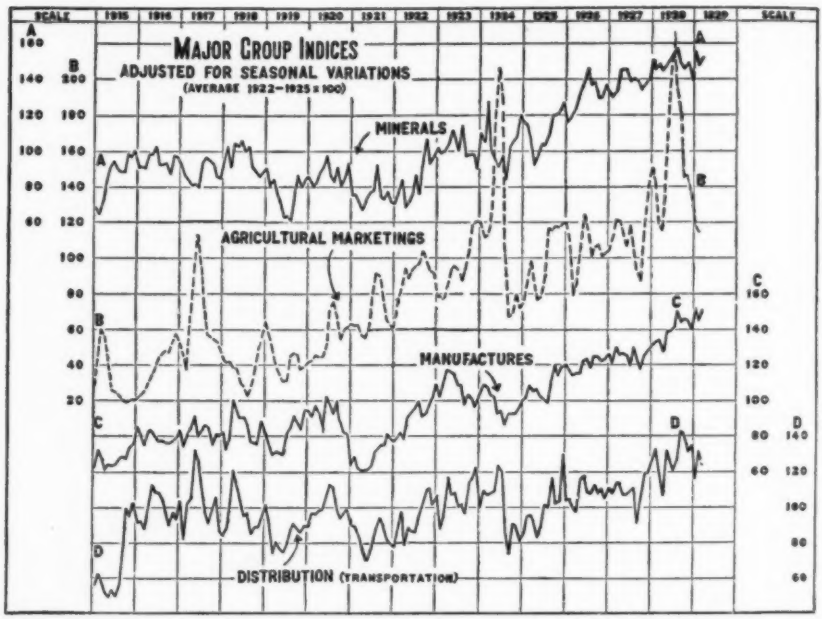
One of the chief obstacles to the early completion of the index numbers was the lack of monthly series for important subjects, particularly for mineral production. In addition, series already available for more recent years had to be carried back. It was finally possible, with the aid of official agencies and private corporations, to calculate the indexes monthly from 1915 to date, and annually back to 1905.

Business activity has been divided into four main groups: (1) mineral production; (2) manufactures; (3) distribution; and (4) agricultural production as it comes into commercial channels through agricultural marketings. The manufactures index includes series representing major industrial groups which for the base period 1922–1925 account for $1,066,000,000, or 83 per cent of the total "value added" by all manufactures of $1,281,000,000. The mineral production group is represented by series which in the base period take in $147,911,000, or 71 per

[1] Mr. R. H. Coats, Dominion Statistician, and his associates have given us invaluable aid in the compilation of data and in matters of fact pertaining to Canadian industry.

cent of a total value of $208,636,000. Distribution is at present represented by transportation alone. It is hoped eventually to include also series more directly representative of wholesale and retail trade; but at present no series such as are available for the United States exist.

CHART I

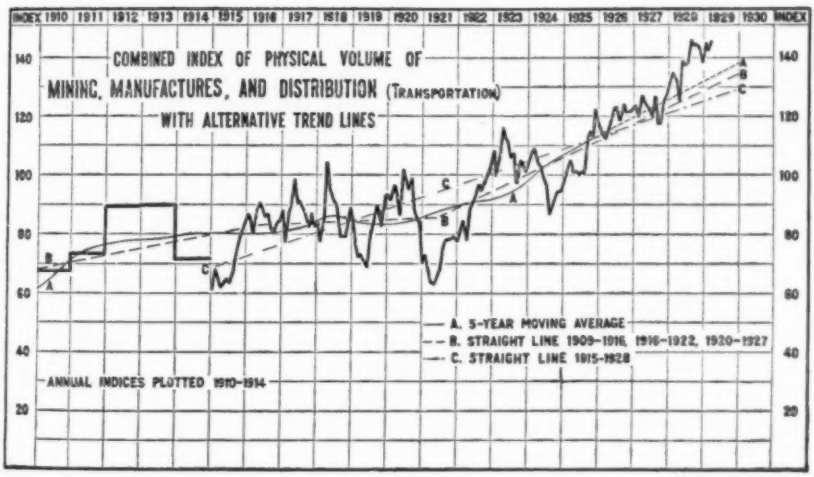


In the composite curve presented in Chart II, the three more nearly homogeneous groups, mineral production, manufacturing, and distribution, are combined. The index of agricultural marketings is presented above for purposes of comparison. All of the series have been adjusted for seasonal variation; and because of its violent fluctuations a three-months moving average of the index of agricultural marketings has been taken.

The great dependence of Canada upon agricultural production is recognized, but the combination of agricultural series with other production data presents theoretical difficulties. Agricultural production, and even agricultural marketings, are not connected with business activity in a simple way. Cyclical variations in physical volume of manufactures (especially when basic series are used), mining, and transportation are generally correlated with increased prices and profits. In agriculture, however, increased production and marketings are often accompanied by falling prices, which may be sufficient to offset greater

production. Thus, farm income may actually decrease with greater physical volume of farm products. The physical volume of manufactures and mining is relatively quick to adjust itself to changing demand; in short, the volume follows the change. On the other hand, greater dependence on natural causes (weather, pests, etc.), the presence of innumerable small producers acting independently, and a relatively inelastic demand for many agricultural products retard economic adjustment. Empirical verification of this is afforded by the extreme variability of the index of agricultural marketings as compared with the other three groups. From a theoretic standpoint, therefore, the inclusion of physical volume series of agricultural production disturbs the economic homogeneity of a general purpose index, such as the Canadian Babsonchart (Chart II).

CHART II


INDEX 1910 1911 1912 1913 1914 1915 1916 1917 1918 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 INDEX
COMBINED INDEX OF PHYSICAL VOLUME OF
MINING, MANUFACTURES, AND DISTRIBUTION (TRANSPORTATION)
WITH ALTERNATIVE TREND LINES
140
120
100
80
60
40
20
A. 5-YEAR MOVING AVERAGE
B. STRAIGHT LINE 1909-1916, 1916-1922, 1920-1927
C. STRAIGHT LINE 1915-1928
ANNUAL INDICES PLOTTED 1910-1914

Production series have been used wherever possible, but for some important industries such data are not available over the entire period or for all branches of the industry. This requires the use, in part, of export or import series of basic materials to represent these industries. In the case of flour, newsprint, rubber manufactures, and sugar, the production series are available for a part of the period. A comparison of such data with imports or exports for overlapping periods has in every case shown marked parallelism. Hence, the change from one series to the other is made by simple substitution of one series for another with an appropriate change in the weighting factors. In the case of lumber and woodpulp exports, no data for comparison of production and imports or exports were available; while in certain other instances

such as imports of petroleum and raw cotton there is little or no domestic production.

No attempt has been made to represent production, as is sometimes done, by storage data, as for example in the case of milk, butter, eggs, cheese. There is little reason in theory for expecting a close similarity of movement between production and storage figures, and this is borne out by the comparisons that have been made. Export series, in every instance studied, more nearly reflect production movement, although in some cases with a decided lag. Employment series, though less sensitive than production data, show a very close relationship to the latter in both cyclical and seasonal movements. Such a series of employment is used for part of the period to get monthly variations for butter and cheese production, available on an annual basis.

Originally it was intended to carry the monthly index back to 1905. However, as quantity data in most cases were available on a monthly basis only for the past five or six years, it was decided to carry the monthly index back only to 1914, for the present study at least, and to use annual data from 1905 through 1914. (For the annual index, see Chart VII on page 138.) To do this it was necessary to obtain through arrangement with individual companies, the Dominion Bureau of Statistics, or departments of the various provinces, the following monthly series, hitherto unavailable:

Coal production for 1915 and 1916
Gold production 1915 to date
Copper production 1915 to date
Lead production 1915–1920
Animal sales 1909–1913
Zinc production 1915 to date
Iron and steel production for 1915 and 1916 and an import index of basic iron and steel products from 1915 to date
Automobile production from 1915 through 1921[1]
Transportation from 1915 through 1920
Tobacco goods entered for consumption

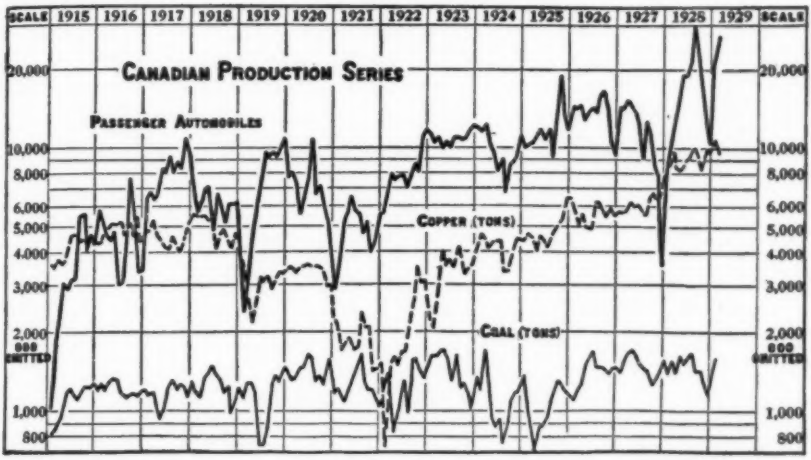
Some of these series, adjusted for seasonal variation, are presented in Charts III and IV. A more detailed consideration of the basic iron and steel imports index, of building, and of the series included in the mineral production group is necessary, since they involve questions of facts and of method.

It was early concluded that to represent production in the iron and steel industry by use of pig iron production and steel ingots and castings output, as is done in most production indexes for the United

[1] Monthly data published in *Survey of Current Business*, United States Department of Commerce, April, 1928, p. 18.

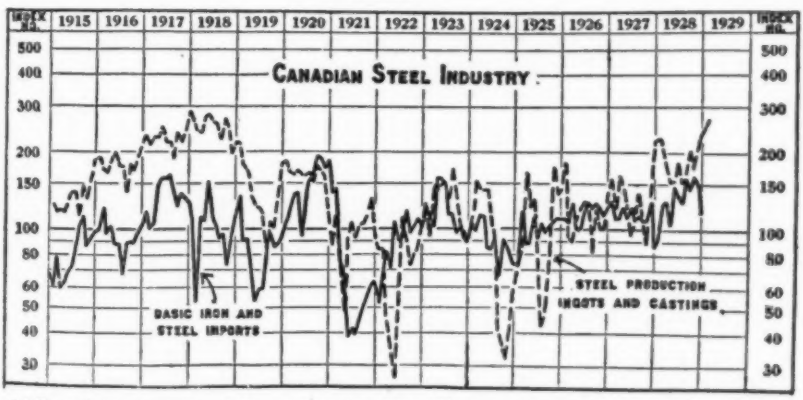
States and Canada, would be inadequate. Large imports of basic iron and steel products, which at later stages are worked into more finished

CHART III



domestic products, make some account of imports indispensable for Canada. In 1924, for example, the value of these imports amounted to $49,038,000, or 146 per cent of the total selling value of domestic production "of pig iron and ferro alloys, steel and rolled products."

CHART IV *



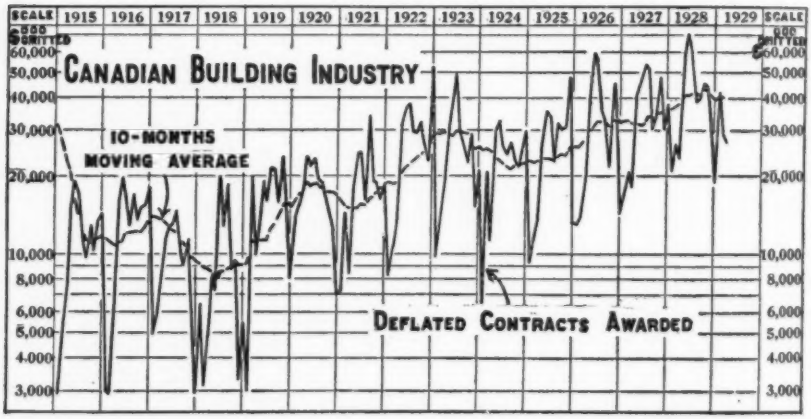
* Adjusted for seasonal variation. 1922–25 = 100. The ten series included in the quantity index of basic iron and steel imports are pig iron, skelp iron, galvanized sheets, iron or steel billets, steel plates, tin plates, iron and steel sheets, and three series of rolled iron and steel hoops, bands, etc.

In the combination of the ten series included in this index (shown above), the same formula and base were used as in the total index

in order to keep the final result logically consistent. The movements of this index frequently differ substantially from, and are sometimes directly opposed to, pig iron and steel ingot and casting production. For example, in the latter half of 1921 and during the early part of 1928, combined production and imports give a much more accurate picture of total activity in this important industry.

To represent the building industry, an index of the physical volume of building at time of actual construction is required. The value of new contracts awarded, as stated in MacLean Building Reports, not

CHART V



only is expressed in dollars but anticipates actual construction activity. Since there is no satisfactory series of the physical volume of building, this value series was first deflated by an index of building costs, a combination of an index of building material prices with one of building wages.[1] Contracts awarded, in physical terms, were spread over the probable construction time by use of an average length of contract activity of ten months. This period was selected after a study of printed sources and correspondence with the industry. A ten-months moving average of the deflated data, centered at the terminal point, is the series finally included in the index. (See Chart V.)

The use of a series so indirectly derived is admittedly open to question. Deflation probably impairs the homogeneity of the index, and

[1] A monthly price index of building materials from 1919 to date is published by the Dominion Bureau of Statistics. This was calculated back to 1913 from the individual series kindly furnished by the Bureau. The building materials prices index was combined with an index of building wages. Since this latter series is on an annual basis, somewhat arbitrary assumptions as to its variations within the year had to be made. The relative importance of the price and wages series, obtained from an official study for proportions in 1924, was adjusted for changes in building material prices and wages respectively. The relative weights were deflated by annual price changes in building materials and in labor respectively. (*Cf.* United States Bureau of Labor Statistics, *Monthly Labor Review*, July, 1925, p. 174.)

the use of ten months as the average period required to complete a contract is approximate. Yet building activity is too important to leave out of account, particularly for Canada, where its influence at the present time is great.

The calculation of the mineral production index offered special difficulties. With the exception of the Dominion Bureau of Statistics series of coal production since 1917 and the American Bureau of Metal Statistics silver and lead series, reported only for the last seven or eight years, monthly data covering substantially all of Canadian production were not available. With the coöperation of the most important producing companies and of various Canadian officials, such series, from 1915 to date, for coal, gold, nickel, silver, copper, lead, and zinc were finally compiled. It is believed that they are fairly representative of total Canadian production of these minerals.

Coal.—Important companies contributed monthly production data for 1915 and 1916. The totals compiled from these individual reports, representing 62 per cent and 70 per cent respectively for the two years, were raised to equal 100 per cent of Canadian annual production, as reported in official sources.

Gold.—Practically 100 per cent of the official Canadian annual production is given by the addition of monthly receipts of gold from Canadian mines at the Royal Mint, Ottawa, and at the Vancouver Assay Office, combined with monthly exports of gold obtained as a direct result of mining operations.

Nickel.—Ontario produces more than 99 per cent of Canadian nickel. Quarterly production figures of nickel-copper matte since 1910 with annual percentages of nickel content were furnished by the Ontario Department of Mines. A monthly series was arbitrarily obtained, as originally suggested by men in the industry, by dividing each quarterly figure for nickel in matte by 3. A study of the quarterly data reveals sufficient uniformity to justify this derivation.

Silver.—Monthly exports were used prior to 1921, when the American Bureau of Metal Statistics series begins. A comparison for the period since 1921 show that exports and production are nearly parallel.

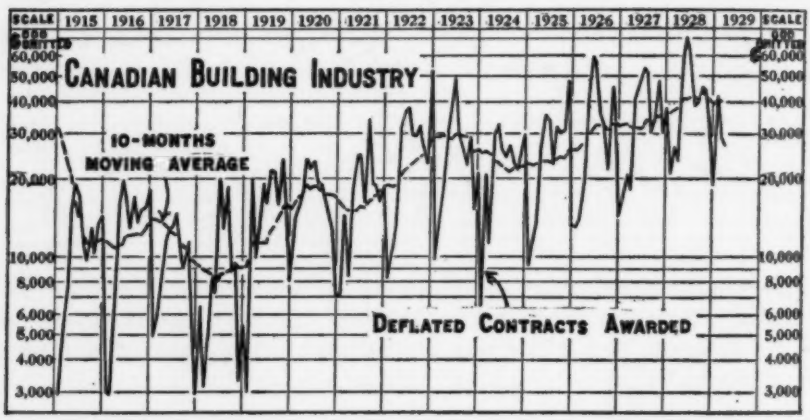
Copper.—Since 1921 this series is a combination of the smelter output reported by the American Bureau of Metal Statistics and the copper content of copper exports in ore, matte, and regulus. Prior to 1921, producing and smelting companies privately reported monthly data for from 88 per cent to practically 100 per cent of the total Canadian output.

Zinc.—This series is obtained by adding together slab zinc production and monthly exports of the zinc content of ores. From 1915 to

in order to keep the final result logically consistent. The movements of this index frequently differ substantially from, and are sometimes directly opposed to, pig iron and steel ingot and casting production. For example, in the latter half of 1921 and during the early part of 1928, combined production and imports give a much more accurate picture of total activity in this important industry.

To represent the building industry, an index of the physical volume of building at time of actual construction is required. The value of new contracts awarded, as stated in MacLean Building Reports, not

CHART V



only is expressed in dollars but anticipates actual construction activity. Since there is no satisfactory series of the physical volume of building, this value series was first deflated by an index of building costs, a combination of an index of building material prices with one of building wages.[1] Contracts awarded, in physical terms, were spread over the probable construction time by use of an average length of contract activity of ten months. This period was selected after a study of printed sources and correspondence with the industry. A ten-months moving average of the deflated data, centered at the terminal point, is the series finally included in the index. (See Chart V.)

The use of a series so indirectly derived is admittedly open to question. Deflation probably impairs the homogeneity of the index, and

[1] A monthly price index of building materials from 1919 to date is published by the Dominion Bureau of Statistics. This was calculated back to 1913 from the individual series kindly furnished by the Bureau. The building materials prices index was combined with an index of building wages. Since this latter series is on an annual basis, somewhat arbitrary assumptions as to its variations within the year had to be made. The relative importance of the price and wages series, obtained from an official study for proportions in 1924, was adjusted for changes in building material prices and wages respectively. The relative weights were deflated by annual price changes in building materials and in labor respectively. (*Cf.* United States Bureau of Labor Statistics, *Monthly Labor Review*, July, 1925, p. 174.)

the use of ten months as the average period required to complete a contract is approximate. Yet building activity is too important to leave out of account, particularly for Canada, where its influence at the present time is great.

The calculation of the mineral production index offered special difficulties. With the exception of the Dominion Bureau of Statistics series of coal production since 1917 and the American Bureau of Metal Statistics silver and lead series, reported only for the last seven or eight years, monthly data covering substantially all of Canadian production were not available. With the coöperation of the most important producing companies and of various Canadian officials, such series, from 1915 to date, for coal, gold, nickel, silver, copper, lead, and zinc were finally compiled. It is believed that they are fairly representative of total Canadian production of these minerals.

Coal.—Important companies contributed monthly production data for 1915 and 1916. The totals compiled from these individual reports, representing 62 per cent and 70 per cent respectively for the two years, were raised to equal 100 per cent of Canadian annual production, as reported in official sources.

Gold.—Practically 100 per cent of the official Canadian annual production is given by the addition of monthly receipts of gold from Canadian mines at the Royal Mint, Ottawa, and at the Vancouver Assay Office, combined with monthly exports of gold obtained as a direct result of mining operations.

Nickel.—Ontario produces more than 99 per cent of Canadian nickel. Quarterly production figures of nickel-copper matte since 1910 with annual percentages of nickel content were furnished by the Ontario Department of Mines. A monthly series was arbitrarily obtained, as originally suggested by men in the industry, by dividing each quarterly figure for nickel in matte by 3. A study of the quarterly data reveals sufficient uniformity to justify this derivation.

Silver.—Monthly exports were used prior to 1921, when the American Bureau of Metal Statistics series begins. A comparison for the period since 1921 show that exports and production are nearly parallel.

Copper.—Since 1921 this series is a combination of the smelter output reported by the American Bureau of Metal Statistics and the copper content of copper exports in ore, matte, and regulus. Prior to 1921, producing and smelting companies privately reported monthly data for from 88 per cent to practically 100 per cent of the total Canadian output.

Zinc.—This series is obtained by adding together slab zinc production and monthly exports of the zinc content of ores. From 1915 to

1922, confidential reports of monthly slab zinc output for virtually 100 per cent of Canadian production takes the place of the American Bureau of Metal Statistics series used currently.

Lead.—For monthly production of lead prior to 1921, when the series of the American Bureau of Metal Statistics becomes available, monthly refined lead output, from 76 per cent to 100 per cent of total Canadian production, has been furnished by the principal producer. To this has been added monthly exports of lead in ore, as in the case of the American Bureau of Metal Statistics series. Refined rather than smelting output has been used, because of substantial duplication in the latter data.

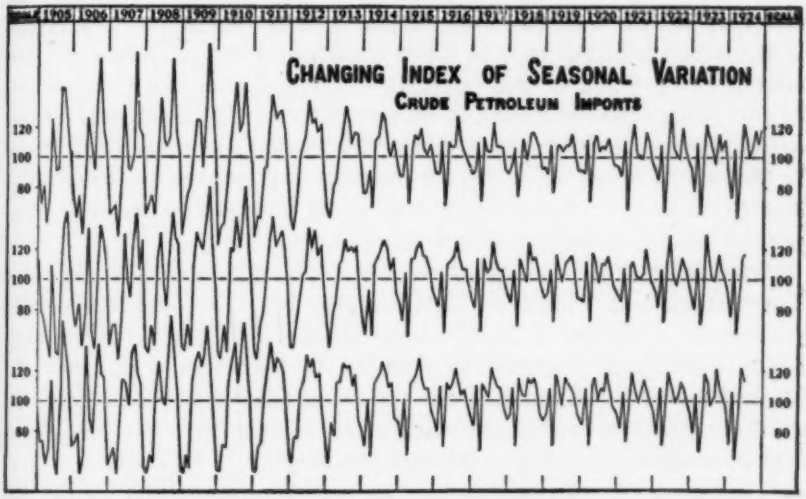
It is difficult to be entirely consistent in the exact stage at which production of the different minerals is measured. Within the limitations of data reported and the technical processes involved, the endeavor has been to measure it as near to the smelter process as possible, for at this stage the metal content is first known with considerable accuracy. Exports of the precious metals probably lag very little behind actual smelting (or refining) production, and their combination with production figures presents little difficulty. The other export series, copper excepted, form a relatively small proportion of total output; and any lag of exports behind smelter output has little effect. In the case of copper, the percentage of exports to total production is in certain years as much as 50 per cent. It has been necessary to take a three-months moving average of the export series to smooth out obviously erratic changes due more to the timing of shipments than to variations in smelter production.

In the adjustment for seasonal variation a comparison, for nearly all of the subjects included, of the individual series adjusted by an index determined on a changing basis with the same series adjusted by use of a constant seasonal index leads to the conclusion that for many series, probably for most, the use of a constant seasonal index is inadequate. In fact, when the latter method is used, the adjusted series over a portion of the period often shows a seasonal movement the inverse of that in the original data. The calculation for a single series of separate constant seasonal indexes for different periods sometimes gives better results. However, the periods of relative homogeneity frequently are too short, and for many series the seasonal effects vary gradually, not abruptly.

In view of these considerations, adjustment of most of the series is made by a changing seasonal index. The method employed is a modification of the methods developed by the United States Federal Reserve Board, Professor L. W. Hall, and others. After considerable

experimentation, a seven-period moving median (arithmetic mean of the middle three items) for each month of the year, of ratios of actuals to a twelve-months moving average is used, both moving means centered at the midpoints of their respective periods. For the extreme years, the curve of this seven-period moving median was extended on the basis of the general trend of the adjacent years.

CHART VI*



*A 5-period moving arithmetic mean, a 6-period moving median (arithmetic mean of middle 2 ratios), and a 7-period moving median (arithmetic mean of middle 3 ratios) were used in obtaining the upper, middle, and lower curves respectively from the ratios of actual data to corresponding ordinates on the 12-months moving average of actuals. The lower seasonal index is used to adjust crude petroleum imports.

The seven-period changing seasonal index gives a smoother seasonal index than either a five-period or a six-period moving median, and requires less extension backwards and forwards on the basis of judgment than a nine-period moving median. The use of a median eliminates the influence of extreme items and the arbitrariness involved in the smoothing of a curve. Hence it is preferred to the Federal Reserve Board's five-period moving arithmetic average; for this when not smoothed allows extreme variations in the original series to influence the moving seasonal index for five years, and when smoothed introduces too much of judgment.

Professor Hall, starting with what appears to be the altogether too flexible definition of seasonal variation as "any change in the data which tends to repeat itself within the period of two years," calculates a three-period moving average, placed at the end of the

period. The result is often an extremely irregular seasonal index, which measures much more than seasonal changes as ordinarily conceived and defined. To be sure, his adjustments give much smoother results, but only by avoiding difficulties, not by solving them.

The use of a changing seasonal index leaves other problems of seasonal measurement still unsolved. Averages, because of extremely variable intensity of seasonal causes and effects, are not always significant. In fact, sometimes it may be best to leave series unadjusted, especially where seasonal variation is greatly obscured by residual fluctuations, as in the case of many export and import series. A belated or an early season or a "normal" growth concentrated in seasonal months, where such variation is extreme, also introduces difficulties yet to be solved. Reducing monthly series to daily average production for the month before adjusting for seasonal changes was not considered feasible, since the information available for most industries was fragmentary and in some instances contradictory.

The "ideal" (Fisher No. 353) formula employed for the United States Babsonchart was considered less suitable for the Canadian study. Apart from the question of its use for more than binary comparisons, ideally the "ideal" formula requires for its most logical and uncomplicated use the fulfilment of the following three conditions: (1) perfect comparability (logical or economic) as between the individual series to be combined; (2) practical exhaustiveness without duplication in subjects included; (3) a system of weights variable on a purely automatic basis, which is virtually impossible without the satisfaction of the first two conditions. Given these conditions, not often, if ever, fully satisfied, the calculation by this formula is straightforward. Lacking these requisites, difficulties arise and compromises must be made.

The use in certain cases of export or import series for all or part of the period to represent production, the combination of some imperfectly comparable series, and the decision to use "value added per unit of production" as a basis for weighting makes necessary the introduction of fixed weights to a much greater extent than in the case of the United States Babsonchart, thus vitiating one of the fundamental benefits of No. 353. Since actual and variable prices are used as weights in No. 353, the constant for any subject is needed not only to give series the weight of the industry but also, where a "value added" basis is adopted, to *reduce* these actual prices to that basis. Furthermore, the constant determined for each subject from a comparison of Census data and the product of actual price and quantity in the fixed weighting base period, must be applied to all four factors (P_oQ_i, P_oQ_o, P_iQ_i, P_iQ_o) of the "ideal" formula if the factor reversal test is to remain satisfied. Since

fixed judgments over relatively long periods cannot be avoided, regardless of formula used, it seems best in the case of Canada, where the three conditions stated in the paragraph above are not nearly satisfied, to choose a formula which does not introduce additional assumptions and factors and in which there is greater flexibility in the introduction of constants.

The basic formula used is the aggregative, $\frac{\Sigma P_o Q_i}{\Sigma P_o Q_o}$, employed by the United States Bureau of Labor, the Federal Reserve Board, Professor Fisher, and others. Its general flexibility and its economic logic adapt it for the calculation of general purpose index numbers. It is free from the statistical bias inherent in averages of relatives, is easy to comprehend, simple to calculate, and makes possible a shift of base by simple division. The weights are fixed nominal prices, the "value added (total value in the case of mineral production and agricultural marketings) per unit of production," as in the case of the new Federal Reserve index of mineral and industrial production. Such nominal prices have not remained the same over the entire period, since actual prices, "value added" by different industries, and the representative character of series have changed. For Canada the rapid industrial development in recent years has involved marked shifts in the relative importance of industries. It is not safe to assume that the same set of constant weights can be used for the entire period without seriously affecting possible comparisons.

In his recent valuable article on the "Construction of Index Numbers," Professor Persons states that "we may be obliged to compute our index number for separate intervals and splice the index numbers in some fashion to get a continuous series." [1] If the separate intervals are too short and change of weighting constants frequent, it becomes more difficult to defend the use of the aggregative as against the "ideal." There also arises the question of how to splice without destroying the homogeneity and continuity of index numbers over a long period.

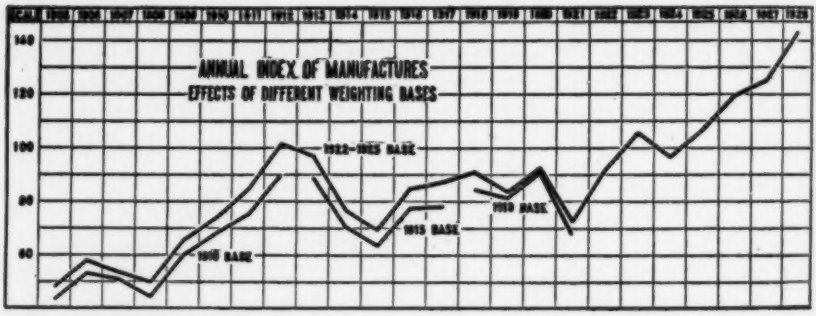
The effect of changing the weighting bases for earlier periods was investigated, first by calculation of annual index numbers and later for the monthly data. The 1910, 1915, and 1919 nominal prices, derived from Census reports, were used for the periods 1909–1912, 1913–1917, and 1918–1921 respectively. In the first two cases the index numbers so calculated are lower by nearly constant per cents than the corresponding index numbers on the 1922–1925 weighting base. Gottfried Haberler [2] has come to the conclusion that for binary comparisons the

[1] "The Effect of Correlation Between Weights and Relatives in the Construction of Index Numbers," *Review of Economic Statistics*, May, 1928, p. 100.

[2] "A New Index Number and Its Meaning," *Quarterly Journal of Economics*, May, 1928, p. 438.

two terms of the "ideal" formula give the upper and lower limits, thus justifying an average of the two. By an extension of this reasoning, the geometric average of the results obtained on the two different weighting bases for each year and month for the above periods was therefore taken. This really gives a modified form of the "ideal" formula, as is pointed out by Woodlief Thomas in the September, 1927, issue of this JOURNAL; or, it may be added, a modified form of the aggregative. Probably this is about as near the truth as we can get and yet make more than binary comparisons possible. Furthermore, this

CHART VII*



*The comparison base is 1922-1925 in every case, the weighting bases only varying.

compromise offers a basis for splicing without too much destroying the homogeneity of the index numbers. It is interesting to note that this compromise formula was adopted independently by the authors and by Woodlief Thomas.

The period 1922–1925 was chosen as the base for comparison. A post-war period for this purpose seems necessary in a growing country where the former levels of production are being permanently left behind. This same period was chosen as the weighting base which remains fixed throughout, as it is the latest period for which "value added" data for manufactures were available. A broader base was not chosen because of the abnormal conditions in 1921. Finally, a recent base period for weighting is desirable to minimize error in the more recent index numbers. However, in time it will again be necessary to take the geometric mean of two indexes—one with the 1922–1925 weightings and the other with the weightings of a later year.

The problem of fitting secular trends to show the "growth factor" and the "normal" for cyclical and residual movements, almost never an easy one, is all the more perplexing for Canada where in the last few years there has occurred a dynamic growth on a relatively small base. The World War, by introducing disturbing forces, further complicates

the problem. Unlike the United States, where these forces raised the level of physical production without very much changing the rate of growth, the war seems to have retarded Canadian business activity—at least in the light of the levels attained in 1911–1913 and in the last few years.

Straight line trends (more complex trends are hardly justified where so few years are available and the situation so non-homogeneous) were fitted to the following periods, among others: 1905–1927; 1915–1928; 1909–1916, 1916–1922, 1920–1927 (see Chart II). A seven-year and a five-year moving average were also calculated as trend lines. The use of the moving average would seem to be in line with the definite tendency to develop moving measures to characterize the changing central tendencies of economic series, as shown by the recently developed seasonal indexes and correlation coefficients calculated on this basis.

Each of these trend lines has some empirical and economic justification; but they give quite different results for the interpretation of current and future business activity. Canadians are prone to claim that the steepest trends are best adapted to the data in the light of probable developments, but it seems difficult to believe that such rates of growth can long continue.

TABLE I

Data Used in Indices of the Physical Volume of Canadian Business

Subjects	Average Net Value 1922-25* (000 omitted)	Relative Weight % of Total	Relative Weight % of Group
Agricultural Marketings	$442,951	0.0	
Cattle Sales	24,398		5.5
Calves Sales	2,088		.5
Hogs Sales	15,308		3.5
Sheep Sales	4,069		.9
Receipts of Wheat	343,376		77.5
Receipts of Oats	19,710		4.4
Receipts of Barley	16,213		3.7
Receipts of Flaxseed	11,077		2.5
Receipts of Rye	6,712		1.5
			100.0
Mineral Production	147,911	9.6	
Coal	60,108		40.6
Gold	29,756		20.1
Nickel	14,977		10.1
‡Silver	12,949		8.8
Copper	11,880		8.0
Lead	12,788		8.6
Zinc	5,453		3.7
			99.9
Manufactures	1,066,306	69.3	
Wood and Paper	*303,369*		*28.5*
†Newsprint Prod. (From '15)	153,402		14.4
Planks & Boards Exports	111,616		10.5
Woodpulp Exports	38,351		3.6
Foodstuffs	*186,181*		*17.5*
†Wheat Fl. Prod. (From '20)	79,500		7.5
Butter & Ch. Prod. ('15, '17–'26) / Cheese Exports ('15, from '26)	31,651		3.0
Foodstuffs (Cont'd)			
†Sugar Mfgd. (From '19)	30,347		2.8
Hogs Slaughtered	17,929		1.7
Cattle Slaughtered	13,070		1.2
Sheep Slaughtered	2,513		.2
Canned Salmon Exports	11,171		1.0
Building	*152,542*		*14.3*
Iron & Steel Products	*152,554*		*14.3*
Steel Ingot Output	76,278		7.2
Pig Iron Production	19,069		1.8
Iron & Steel Imports	57,207		5.4
Textiles	*149,808*		*14.0*
Cotton — Raw — Imports	74,904		7.0
Cotton — Yarn — Imports	37,452		3.5
Wool — Raw — Imports	18,726		1.8
Wool — Yarn — Imports	18,726		1.8
Tobacco	*40,791*		*3.8*
Cigarettes	20,716		1.9
Cigars	10,358		1.0
Tobacco & Snuff	9,717		.9
Automobiles & Accessories	*35,443*		*3.3*
Passenger Car Production	35,443		3.3
Rubber Goods	*32,538*		*3.1*
Petroleum Products	*13,080*		*1.2*
Petroleum — Crude — Imports	13,080		1.2
Distribution	325,049	21.1	100.0
Net Ton Miles Freight	325,049		100.0

(Current figures computed by the Babson Statistical Organization for Gold, Copper, Zinc, Building, and Rubber Goods; by Department of Mines, Ontario, for Nickel; by Am. Bureau of Metal Statistics for Silver and Lead; by Newsprint Service Bureau for Newsprint; by Dept. of National Revenue for Tobacco. All other series are from the Dominion Bureau of Statistics.)

*Total value used in Mineral Production and in Agricultural Marketings.

†Exports or imports substituted prior to years indicated. ‡Silver exports used prior to 1921.

†Exports or imports substituted prior to years indicated. ‡Silver exports used prior to 1921.

TABLE II

INDICES OF THE PHYSICAL VOLUME OF CANADIAN BUSINESS
Adjusted for Seasonal Variations

Combined Index of Mining, Manufactures, and Distribution

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	61.5	86.6	89.0	84.1	86.4	93.4	69.9	76.5	108.4	103.5	98.8	116.1	123.7	131.9	144.8	Jan.
Feb.	69.3	85.2	77.4	77.1	78.1	91.3	73.0	81.0	98.6	108.4	100.9	114.4	119.0	134.4	142.9	Feb.
Mar.	66.3	79.7	87.4	84.6	71.7	96.6	67.1	85.3	106.4	109.1	105.3	112.1	126.8	132.5	145.3	Mar.
April	61.6	87.6	88.5	104.3	73.3	93.5	63.8	77.4	116.2	104.0	101.1	112.6	124.5	125.1		April
May	63.2	91.1	99.1	96.1	71.3	86.2	62.7	88.1	113.4	103.2	100.6	117.2	123.7	138.8		May
June	64.7	89.1	90.3	92.8	69.0	101.6	64.7	90.0	112.3	98.8	99.7	122.7	122.7	137.5		June
July	63.8	85.7	91.1	91.2	78.7	99.4	67.3	92.3	105.6	95.7	101.1	122.5	119.0	138.0		July
Aug.	67.1	87.2	89.4	89.7	82.5	94.8	76.4	96.7	107.3	85.5	99.9	117.7	126.9	145.9		Aug.
Sept.	72.5	81.9	86.9	79.2	89.8	99.4	78.3	93.6	96.9	89.2	112.2	123.6	116.9	144.3		Sept.
Oct.	77.7	80.6	81.8	79.2	87.4	88.8	77.8	97.2	104.7	93.3	115.3	121.0	117.0	144.7		Oct.
Nov.	80.6	83.8	87.0	78.8	83.2	85.5	78.4	99.9	103.8	93.5	111.7	120.8	125.2	143.6		Nov.
Dec.	84.0	84.3	81.8	90.2	93.0	85.1	79.1	100.6	101.2	94.4	122.0	120.5	126.9	138.6		Dec.
Yearly Av.	69.4	85.2	87.5	87.3	80.4	93.0	71.5	89.9	106.2	98.2	105.7	118.4	122.7	139.0		Yearly Av.

Agricultural Marketings (*3 months moving average*)

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	27.7	21.2	53.3	42.6	64.1	40.6	62.7	60.1	78.1	119.3	75.4	118.8	103.1	150.3	117.1	Jan.
Feb.	43.2	22.8	46.3	40.5	56.0	41.5	61.6	77.4	77.5	111.0	85.5	98.7	109.6	142.9	115.3	Feb.
Mar.	60.4	25.1	38.1	42.2	49.8	45.3	61.8	80.3	75.7	112.1	97.5	78.8	120.6	118.4		Mar.
April	55.6	29.0	66.8	37.8	39.7	43.5	55.8	94.2	86.2	119.6	87.1	85.5	120.4	115.2		April
May	42.2	33.1	93.8	38.1	33.6	43.5	54.7	88.0	93.7	181.2	75.6	111.7	111.8	160.6		May
June	25.9	36.9	113.3	30.0	31.3	50.0	60.3	92.9	94.6	205.9	78.4	124.2	106.0	192.9		June
July	24.8	43.7	94.6	27.1	31.3	70.8	81.8	95.1	92.8	187.4	88.7	119.7	117.9	225.9		July
Aug.	22.6	45.8	70.8	21.6	44.5	74.7	90.6	96.3	87.4	102.6	116.1	100.3	103.3	196.2		Aug.
Sept.	20.4	47.8	57.4	27.2	46.4	67.9	89.4	102.7	93.3	66.6	115.2	105.4	93.4	183.3		Sept.
Oct.	19.1	47.1	55.3	34.8	45.2	53.7	73.9	97.9	104.0	68.3	116.6	105.8	86.5	145.0		Oct.
Nov.	19.8	52.7	54.1	47.6	37.3	59.8	64.8	93.4	118.2	80.3	115.7	99.7	115.1	146.4		Nov.
Dec.	20.3	58.4	49.6	54.5	37.8	60.6	61.8	90.6	120.0	72.2	119.0	102.4	138.9	133.7		Dec.
Yearly Av.	31.9	38.6	66.3	36.9	42.9	54.8	67.9	88.8	94.2	118.8	98.2	102.6	110.0	161.5		Yearly Av.

TABLE II—*Continued*

Mineral Production

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	67.8	89.6	91.8	96.3	89.9	85.3	74.9	69.7	102.2	110.0	115.5	115.5	133.0	151.1	155.0	Jan.
Feb.	64.1	91.0	86.2	102.2	79.9	79.7	74.8	77.4	99.0	105.3	114.6	117.5	130.4	144.7	147.5	Feb.
Mar.	71.3	90.1	82.8	91.2	84.0	81.2	70.6	84.0	100.1	126.8	106.4	121.7	133.3	148.2		Mar.
April	83.1	98.4	80.1	104.3	77.0	88.0	65.8	68.1	104.4	99.6	92.2	127.5	144.5	143.7		April
May	90.4	97.7	81.1	102.2	69.9	90.7	72.8	71.1	112.2	94.5	98.4	133.9	145.4	147.6		May
June	94.0	101.7	78.5	105.5	61.8	96.7	76.4	75.1	107.2	91.3	104.1	139.4	144.7	150.7		June
July	90.2	92.2	92.4	98.5	62.6	86.0	76.6	85.9	99.9	97.7	103.4	146.4	138.1	154.8		July
Aug.	87.9	91.5	95.9	101.9	60.0	81.7	92.1	76.2	114.9	84.3	109.7	136.8	140.2	156.9		Aug.
Sept.	88.2	93.3	93.9	91.3	75.5	90.7	74.1	94.9	96.8	102.4	120.1	139.0	139.2	150.0		Sept.
Oct.	97.8	87.4	91.5	87.9	85.5	79.5	72.2	106.8	98.1	105.1	119.6	129.8	134.1	146.0		Oct.
Nov.	95.9	97.0	84.9	84.9	78.6	85.2	76.5	93.4	97.6	108.2	121.1	130.1	137.3	148.5		Nov.
Dec.	100.3	96.2	83.7	87.5	84.1	93.4	70.6	96.9	90.1	119.5	127.3	137.0	140.1	139.3		Dec.
Yearly Av.	85.9	93.8	86.9	96.1	75.7	86.5	74.8	83.3	101.9	103.7	111.0	131.2	138.4	148.5		Yearly Av.

Distribution (Transportation)

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	55.5	90.9	104.0	85.4	102.4	89.0	89.8	77.8	107.3	100.1	85.8	103.7	112.4	127.9	116.0	Jan.
Feb.	63.2	92.3	82.1	87.7	90.8	96.9	88.6	88.0	88.4	109.8	94.6	105.2	107.8	133.3	132.4	Feb.
Mar.	59.1	87.7	102.0	104.2	74.3	95.9	85.0	97.6	96.3	106.8	96.4	100.7	113.5	119.6	124.4	Mar.
April	52.1	99.8	105.1	120.9	81.3	98.2	77.8	79.2	116.6	107.8	92.8	97.9	113.5	106.8		April
May	48.7	113.2	131.8	111.4	76.7	99.1	70.1	88.5	106.6	109.0	83.4	116.3	107.4	133.2		May
June	54.0	109.0	126.9	102.0	74.6	106.3	72.8	87.3	108.2	123.2	90.3	117.7	106.6	126.6		June
July	50.2	107.8	113.2	95.4	79.6	112.6	80.9	85.6	99.9	100.2	102.4	109.3	109.4	120.6		July
Aug.	56.1	114.5	97.0	96.6	86.9	111.3	88.2	94.5	102.2	84.5	99.9	108.2	111.2	126.9		Aug.
Sept.	77.5	95.6	90.6	84.9	90.9	98.7	94.0	101.2	97.2	73.7	116.5	112.8	90.6	142.9		Sept.
Oct.	100.1	90.0	98.4	89.1	87.9	93.5	90.4	110.3	113.8	91.3	102.4	108.4	107.5	142.0		Oct.
Nov.	95.3	96.6	105.9	88.8	86.0	96.8	81.6	111.3	115.9	90.0	102.6	110.8	119.8	136.9		Nov.
Dec.	102.8	92.8	86.6	94.8	90.1	98.5	80.3	100.5	123.4	81.2	130.0	104.9	119.6	134.3		Dec.
Yearly Av.	67.9	99.2	103.6	96.8	85.1	99.7	83.3	93.5	106.3	98.1	99.8	108.0	109.9	129.2		Yearly Av.

TABLE II—*Concluded*

Manufactures

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	62.2	84.5	83.4	82.2	81.9	95.3	64.2	77.0	109.4	103.5	100.4	119.9	125.7	130.3	152.1	Jan.
Feb.	72.0	81.7	74.3	71.4	74.7	91.1	62.7	79.1	101.5	108.2	100.8	116.6	120.7	133.1	145.3	Feb.
Mar.	67.7	75.1	81.4	78.8	69.9	98.4	62.1	81.5	110.2	107.2	107.8	114.1	129.8	134.2	150.6	Mar.
April	61.1	81.6	84.1	100.0	70.9	92.7	60.0	78.0	117.4	103.2	104.7	115.0	125.0	128.0		April
May	63.5	82.7	90.8	91.4	70.0	82.3	59.5	90.2	115.5	102.4	106.1	114.9	125.6	139.1		May
June	63.3	80.2	79.8	88.8	68.5	100.7	61.3	92.7	114.1	92.0	101.9	121.7	124.5	138.8		June
July	63.9	76.6	83.4	89.1	80.3	97.5	62.7	95.2	107.9	93.8	100.2	123.1	119.1	140.9		July
Aug.	67.2	77.1	85.7	86.3	84.0	92.1	71.5	100.0	107.6	85.8	98.3	117.9	129.8	150.1		Aug.
Sept.	68.1	75.6	84.4	76.6	90.9	100.3	74.8	90.9	96.6	92.0	109.6	124.6	121.9	143.7		Sept.
Oct.	66.8	76.1	74.5	75.6	87.3	88.5	75.2	91.6	102.5	92.2	118.5	123.5	117.4	145.5		Oct.
Nov.	73.1	77.2	80.8	75.4	82.8	82.6	81.4	97.1	100.7	92.4	113.0	122.4	125.0	144.7		Nov.
Dec.	74.8	79.2	79.6	89.1	94.7	80.6	79.7	101.0	95.6	94.9	118.5	123.0	127.2	139.6		Dec.
Yearly Av.	67.0	79.0	81.8	83.7	79.7	91.8	68.4	89.5	106.6	97.3	106.6	119.7	129.3	137.9		Yearly Av.

Index of Iron and Steel Industry

Year	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	Year
Jan.	93.5	148.4	177.4	181.8	179.5	149.8	109.7	70.9	120.8	114.3	72.3	150.5	142.3	165.5	206.3	Jan.
Feb.	101.1	150.4	159.8	151.4	139.9	142.2	126.5	76.5	102.6	131.2	93.8	101.1	117.0	173.7	217.4	Feb.
Mar.	93.9	134.5	163.6	173.4	133.4	146.4	84.6	63.2	125.8	130.9	128.0	103.3	138.8	154.2	219.5	Mar.
April	93.0	148.4	186.8	189.8	110.5	152.8	60.8	52.2	147.7	130.3	105.4	105.6	135.8	135.2		April
May	100.8	145.7	200.9	211.4	93.6	132.7	69.5	58.7	152.4	118.7	120.1	109.8	119.6	148.9		May
June	108.3	139.0	185.6	186.9	93.2	150.3	77.4	67.8	152.0	98.2	94.8	122.8	108.0	162.9		June
July	115.4	126.8	187.8	183.9	89.7	159.7	69.8	97.3	127.9	94.7	66.2	126.3	105.5	150.6		July
Aug.	110.8	118.3	165.7	158.7	84.9	157.0	77.3	99.2	153.2	51.5	66.1	101.5	128.7	163.7		Aug.
Sept.	132.7	141.3	180.2	181.9	103.0	185.3	79.9	80.5	122.3	58.9	85.4	124.1	109.7	179.6		Sept.
Oct.	112.3	134.9	177.0	164.1	93.1	180.9	86.6	86.8	107.5	55.2	143.7	111.6	94.0	167.4		Oct.
Nov.	127.1	146.4	183.1	150.3	114.6	169.3	96.2	91.2	101.3	57.3	127.8	106.6	133.3	187.7		Nov.
Dec.	144.8	163.9	203.3	171.2	139.2	145.5	79.1	98.1	97.4	64.8	130.2	129.9	162.6	178.5		Dec.
Yearly Av.	111.1	141.5	181.3	167.1	107.1	156.0	84.8	78.5	125.9	83.8	102.8	116.1	124.6	163.7		Yearly Av.

THE USE OF ISORROPIC LINES IN DETERMINING REGRESSION SURFACES

Illustrated by a Study of the Relationship of Maine Potato Yields to Early and Late Rainfall

By Frederick V. Waugh, *United States Bureau of Agricultural Economics*

An isorropic line in a diagram is defined as the locus of all points for which a specified function has a constant value. In Charts I and II regression surfaces are described by such isorropic lines; that is, the lines numbered 300; 290; etc., locate on the regression surfaces constant values of X_1 as determined by $X_1 = a + b_{12.3}X_2 + b_{13.2}X_3$ and by $X_1 = f(X_1 X_2)$. Isorropic lines are useful not only in presenting the results of three-dimensional correlation, but also in the determination of the regression surface. The use of these lines is a modification of the method of analysis described by Dr. Ezekiel in this Journal.[1] The following discussion of the joint relation of Maine potato yields to early and late rainfall will illustrate the method used.

Recent studies of New England potato yields have demonstrated two important facts. The first is that farmers' estimates of the condition of the potato crop on July 1 and on August 1 give little, if any, indication of probable yields. The second is that rainfall has been so closely related to potato yields in the New England states that most of the year-to-year variation in yields during the past fifteen years can be attributed to this factor and to a gradual upward trend in yields.

The reason farmers are unable to estimate the potato crop accurately early in the season is perhaps evident. They must judge the prospects on the appearance of the vines rather than on tuber development. Heavy rainfall produces good vine growth but low yields. The result is a minus correlation between August 1 reported condition and final yield in Maine. On the other hand, by August 1 the yield of potatoes in Maine appears to be determined within fairly close limits by weather conditions prior to that date. The coefficients of correlation between Maine potato yields and the two factors of rainfall and trend for the years 1913 to 1927 are: using rainfall from May 1 to June 30, 0.774; using rainfall from May 1 to July 31, 0.903; using rainfall from May 1 to August 31, 0.903; and using rainfall from May 1 to September 30, 0.906.[2]

[1] M. J. B. Ezekiel, "The Determination of Curvilinear Regression Surfaces in the Presence of Other Variables," this Journal, Vol. XXI, September, 1926, p. 310.

[2] The rainfall series used in this study is a weighted average of reports from five stations distributed among the potato producing sections of the state. In calculating the above correlation coefficients,

It will be noted that the correlation using rainfall from May 1 to July 31 is practically as high as the correlations including August and September rainfall. Similar results are obtained for the other New England states. In spite of the failure of August and September rainfall to increase the correlation in any significant degree, it is the opinion of potato growers that rainfall conditions during these months have in some years had a decided influence on yields. Fair prospects in midsummer have resulted in high, medium, or low yields, depending on rainfall during the last two months of the growing season.

It is probable that in studying only the total rainfall from May 1 to the first of each month, the relationship of rainfall to yields is not accurately expressed. However, when rainfall during the season is divided into two parts—May 1 to July 31 and August 1 to September 15—there is still no significant improvement in the coefficient of multiple correlation. The relationships discussed so far have all been linear (in the form $X_1 = a + b\,X_2 + cX_3$), where X_1 represents final yield, X_2 represents total rainfall, and X_3 represents trend. If the relationships are expressed as second approximation curves (such as $X_1 = a + f(X_2) + f(X_3)$, the correlations are all improved, but rainfall from May 1 to July 31 still seems to offer almost as good an explanation of yields as does either total—May 1–September 15 rainfall or separate early and late rainfall.

A reconsideration of the problem led to the belief that the true relationship of yields to early and late rainfall is expressed neither by a formula of the type, $X_1 = a + b\,X_2 + c\,X_3$, nor by the more generalized expression, $X_1 = a + f\,(X_2) + f\,(X_3)$, but that it might be expressed accurately in the form $X_1 = f\,(X_2, X_3)$. That is, early and late rainfall probably should not be treated as separate, independent factors influencing yields, but should be studied jointly. The average influence of late rainfall on yields may mean little. A wet late season following a wet early season may seriously decrease yields, while a wet late season following a dry early season may increase yields. If this is true, the real relation between yields and early rainfall and late rainfall cannot be expressed either by straight lines or by curves, but only by a surface which indicates the yields which result from the various possible combinations of early and late rainfall.

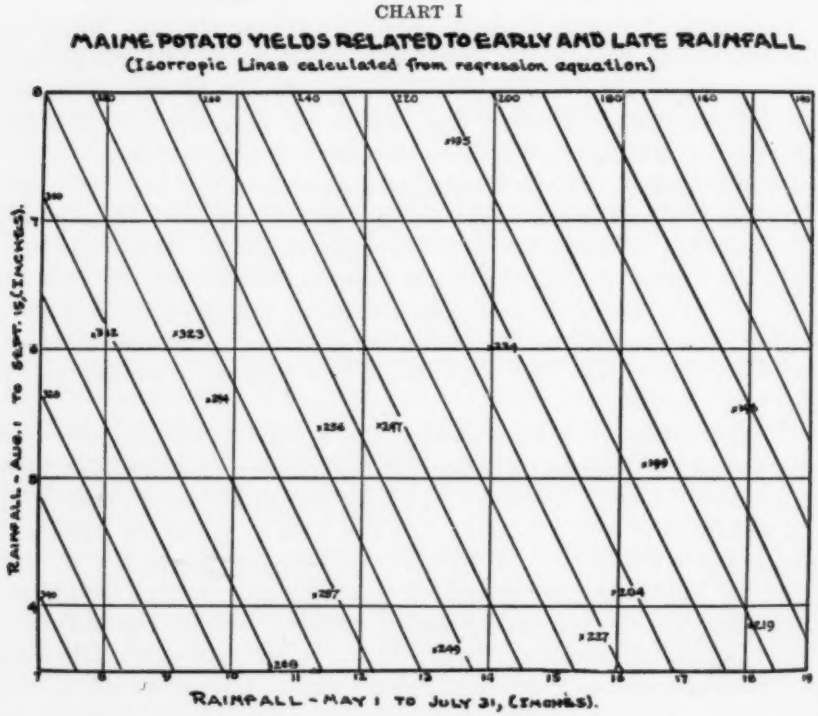
The method by which such a curvilinear regression surface was determined is very similar to the method of determining curvilinear correlations which has been presented in this JOURNAL by Dr. Ezekiel.[1]

rainfall during the month of July was doubled, since a preliminary analysis of the data indicated that an additional inch of rainfall during that month was associated with about twice as great a decrease in yields as an additional inch in the other months.

[1] M. J. B. Ezekiel, "A Method of Handling Curvilinear Correlation for any Number of Variables," this JOURNAL, Vol. XIX, December, 1924, p. 444.

This method involves the plotting of net regression lines for each independent factor, plotting the residuals of the dependent factor around the regression lines, and approximating these residuals with free-hand curves. In the solution of the present problem the only modification of this method was that the combined regression coefficients of two independent factors were plotted as one surface, residuals were plotted around this surface, and these residuals were approximated by a free-hand curvilinear surface.

CHART I


MAINE POTATO YIELDS RELATED TO EARLY AND LATE RAINFALL
(Isorropic Lines calculated from regression equation)
RAINFALL - AUG. 1 TO SEPT. 15, (INCHES).
RAINFALL - MAY 1 TO JULY 31, (INCHES).

Instead of plotting two regression lines, $X_1 = a + b_{12.3}X_2$ and $X_1 = a + b_{13.2}X_3$, a regression surface was plotted representing the formula, $X_1 = a + b_{12.3}X_2 + b_{13.2}X_3$. Chart I shows the regression surface resulting from a straight line multiple correlation analysis of the data in the accompanying table. This analysis results in the regression equation $X_1 = 438.482414 - 12.897356\,X_2 - 12.830652\,X_3 + 2.935037\,X_4$. The diagonal lines across Chart I show the various combinations of early and late rainfall which in 1927 ($X_1 = 15$) would have been expected to result in yields of 160, 170, 180, etc., bushels per acre as calculated from the above formula. These isorropic lines or lines of

equal yield, numbered at the top of the chart, indicate that an average yield of about 300 bushels per acre in 1927 [1] should, theoretically, have resulted from early rainfall of 7.5 inches followed by late rainfall of 6.75 inches; or from early rainfall of 8.5 inches followed by late rainfall of 5.75 inches, or by early rainfall of 9.5 inches followed by late rainfall of 4.75 inches.

MAINE POTATO YIELDS: RELATION TO EARLY AND LATE RAINFALL

Year	Rainfall		Trend	Yields	Yields estimated from regression equation*	Residual	Yields estimated from regression surface†	Residual
	May 1 to July 31 X_2	Aug. 1 to Sept. 15 X_3	X_4	X_1	X_1	z_1	X_1	z_2
1913.....	13.17	3.66	1	220	224.6	− 4.6	225	− 5
1914.....	11.33	4.08	2	260	245.9	+14.1	238	+22
1915.....	15.96	4.12	3	179	188.6	− 9.6	195	−16
1916.....	15.46	3.77	4	204	202.5	+ 1.5	200	+ 4
1917.....	17.77	5.53	5	125	153.0	−28.0	128	− 3
1918.....	18.09	3.87	6	200	173.1	+26.9	189	+11
1919.....	12.25	5.41	7	230	231.6	− 1.6	232	− 2
1920.....	13.29	7.62	8	177	192.8	−15.8	177	0
1921.....	7.82	6.11	9	298	285.6	−12.4	300	− 2
1922.....	16.40	5.12	10	187	190.6	− 3.6	196	− 9
1923.....	10.61	3.51	11	258	288.9	−30.9	260	− 2
1924.....	9.10	6.13	12	315	277.7	+37.3	314	+ 1
1925.....	11.35	5.38	13	250	261.2	−11.2	254	− 4
1926.....	9.60	5.60	14	290	283.9	+ 6.1	291	− 1
1927.....	13.98	6.02	15	232	225.0	+ 7.0	223	+ 9

* Straight-line regression equation
$z_1 = 438.482414 - 12.897356\ x_2 - 12.83065\ x_3 + 2.9350337$
$R_{1.234} = .896567$

† Curvilinear regression surface
$R_{1.234} = .985172$

It will be noted that the isorropic lines on the chart are straight lines, equidistant and parallel. In other words, the surface described by the formula is a plane. It is not a level surface (which would indicate no relationship between rainfall and yield) but a plane which is tilted from the high point in the lower left-hand corner to the low point in the upper right-hand corner. The relation of one dependent factor to two independent factors, as expressed by $X_1 = a + b\ X_2 + cX_3$, can always be described by a plane, because of the fact that b and c are constants. The isorropic lines in Chart I, for example, representing differences of ten bushels in probable yields, are each at a distance from the next line of $\frac{10}{b_{12.3}}$ on the X-axis and of $\frac{10}{b_{13.2}}$ on the Y-axis. In the regression equation the constant a determines the height of this plane above the base line, while the constants, $b_{12.3}$ and $b_{13.2}$ determine the slope of the plane measured on two axes at right angles to one another.

If the equation used to determine the regression surface is of the more generalized type $(X_1 = a + f\ (X_2) + f\ (X_3)$, the surface described may not

[1] The yield has been adjusted for trend in order to give 1927 values rather than mean values.

be a plane, since the functions of X_2 and X_3 may not be constants. Nevertheless, the surface described by such an equation is limited in somewhat the same manner as is the plane described by $X_1 = a + b X_2 + c X_3$. The equation explains all values of X_1 by the addition of functions of X_2 and X_3 and a constant. The addition of these functions determines that as X_3 increases from the lowest to the highest observed value, the shape of the curve, $f(X_2)$, remains unchanged, merely being raised or lowered in relation to the base line. Similarly the shape of the curve, $f(X_3)$, remains unchanged as X_2 increases from the lowest to the highest observed value. That is, the partial derivatives with respect to each variable are constant. Since on the surface representing the relation of X_2 and X_3 to X_1 the curves $f(X_2)$ and $f(X_3)$ are lines drawn through any point at right angles to the axes, it will be seen that such lines (reverting to the technical language of surveying these would be called "profiles") must all be parallel to one another.

The potato yields in Maine from 1913 to 1927 (adjusted for trend according to the net regression on time) are plotted in Chart I. These yields are located on the chart in respect to both the X and the Y axes, representing early and late rainfall. Each dot on the chart shows for one year the amount of early and late rainfall which fell in Maine. Beside each dot is written the average yield of that year. Thus, at the top, left-hand corner of the chart is located a dot at 7.82 measured on the X-axis and 6.11 measured on the Y-axis. The dot is labeled 312. This indicates that during one season there were 7.82 inches of rainfall during the period from May 1 to July 31 and 6.11 inches of rainfall from August 1 to September 15, resulting in an average yield (adjusted for trend) of 312 bushels.

The yield figures on this chart are similar to elevations on a map. The result is an uneven surface with many small peaks and valleys. The problem of determining a "regression surface" is that of smoothing out these peaks and valleys without changing the general shape or slope of the surface. It is the same problem the surveyor has in grading an irregular surface of land. Instead of smoothing in one direction as in the case of the usual problem in curvilinear correlation, it is necessary to smooth in two directions simultaneously. In such two-way smoothing the statistician can make good use of a method commonly used in surveying—the contour line. Contour lines, or lines of equal elevation, are used on topographical maps to describe the existing slope or shape of a surface of land. If the surface of a plot of land is to be graded (*i. e.*, "smoothed"), the engineer or the landscape architect first calculates the elevation of the new surface by smoothing the

existing contour lines. It is possible to use a similar method of analysis in smoothing a "regression surface."

In the case of the problem presented here it is doubtful whether either the equation $X_1 = a + b\ X_2 + cX_3$ or the equation $X_1 = a + f(X_2) + f(X_3)$ can accurately describe the relation of yields to early and late rainfall. If either of these equations, for example, indicated that late rainfall 20 per cent in excess of normal was associated on the average with a decrease of 50 bushels per acre in normal yields, it would be interpreted to mean that such a decrease in yields would be likely to occur whether the heavy late rainfall followed a dry or a wet early season. But observation tells us that heavy rainfall in August and September may be beneficial to the potato crop if the previous season has been dry, while it may be disastrous if the previous season has been wet. That is, there is reason to think that actually the curve represented by $f(X_3)$ changes shape and even direction, depending on the values of X_2 with which it is associated.

With this in mind, let us reëxamine Chart I. A group of dots to the left of the chart may be selected to represent yields in years of light rainfall during the period May 1 to July 31. Another group on the right-hand side of the chart may be selected to represent yields in years of heavy early rainfall. Studying the relation of yields to late rainfall in these two groups leads to opposite conclusions. Following light rainfall in the early season, rather heavy late rainfall seems to benefit the crop, while heavy late rainfall following heavy early rainfall results in decidedly lower yields. It is, perhaps, evident that the plane surface described by the isorropic lines on Chart I does not offer an adequate explanation of the observed relation of early and late rainfall to yields.

The observed relation can be approximated by twisting the isorropic lines in Chart I in order to warp the original plane surface and to make it fit the residuals more closely. This presents no difficulty to the man who has been trained in surveying. The important thing to remember is that not only should each isorropic line be a regular, smooth curve, but each line should be so related to each other line that the whole surface will be smooth. When such lines have been tentatively drawn, it is well to read off the estimated values of X_1 along several lines at right angles to the X_2 and X_3 axes. The values along these lines (or profiles) can then be smoothed and the corrected values replotted. This process assures the regularity or smoothness of the surface. The resulting isorropic lines define the first approximation of a regression surface.

In case of the present study, a first approximation curve $f(X_4)$

was also drawn representing the net trend of yields. The first approximation was then complete, including a net first approximation surface showing the relationship of X_1 (or yields) to X_2 and X_3 (or early and late rainfall) and also a net first approximation curve showing the relationship of X_1 (or yields) to X_4 (or trend). Values of X_1 were next read from the surface ($X_1 = f(X_2X_3)$) and from the curve ($X_1 = f(X_4)$). A second set of residuals was calculated from these values and was plotted on charts. From these residuals second approximations of the net surface and the net curve were made.

CHART II

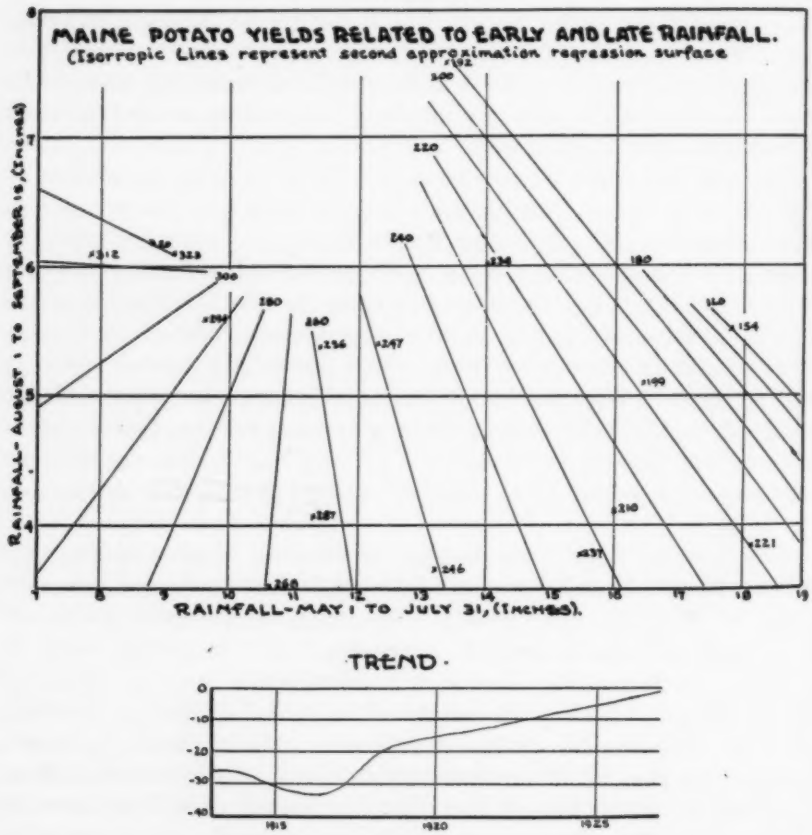

MAINE POTATO YIELDS RELATED TO EARLY AND LATE RAINFALL.
(Isorropic Lines represent second approximation regression surface
RAINFALL-AUGUST 1 TO SEPTEMBER 15, (INCHES)
RAINFALL-MAY 1 TO JULY 31, (INCHES).
TREND.

Chart II shows the second approximations in the equation $X_1 = f(X_2X_3) + f(X_4)$. This results in a coefficient of correlation of 0.985 and a standard error of estimate of 8.6 bushels. The standard deviation of potato yields in Maine during the same years was 50.0 bushels.

An examination of the "lines of equal yield" in Chart II will show, first, that the surface described is smooth and regular in spite of being decidedly warped and, second, that the surface fits the observations much more accurately than does the plane surface on Chart I calculated from the straight line regression coefficients.

By projecting the analysis into three dimensions rather than the customary two dimensions, it is possible to understand more clearly and more accurately the relationship of yields to early and late rainfall. It is demonstrated that late rainfall has had, in certain years, a decided influence on yields, but that this influence cannot be accurately described in terms of two dimensions. Heavy late rainfall may be a blessing or a curse, depending on the amount of rainfall which has fallen prior to August 1. In stating the relationship of late rainfall to yields either as $X_1 = a + b\ X_2$, or as $X_1 = a + f\ (X_2)$, we arrive at an average relationship for all years, wet or dry, but on any particular year the true relationship may be quite different.

The use of isorropic lines, or lines of equal value, is a modification of Dr. Ezekiel's method which makes it possible to smooth residuals in two directions simultaneously. This modification is particularly useful in the analysis of time series because of the limited number of observations and the consequent difficulty in cross-tabulating and averaging the observations. Dr. Ezekiel's method is similar to the method of grading a surface of land by determining a series of "profiles," while the modification of the method used here is similar to the method of grading in one process by contour lines.

STATISTICS AND ECONOMIC FORECASTING

By E. M. Burns, *Columbia University*

In the course of his recent book[1] investigating the methodology and the possibilities of economic forecasting, Dr. Morgenstern, of Vienna, challenges the claims of statisticians to contribute to this end. He concludes that no basis for economic forecasting[2] is to be found either in the nature of our knowledge of the behavior of the economic world or in the manipulation of statistical material as a convenient summary of economic quantities or as data for probability calculations. Of the purely economic aspects of this book, which provides a much needed and skillful analysis of the central problems of theory at issue, little will be said, although grounds will be suggested for rejecting some of the conclusions. But those sections of his work which analyze the economic meaning of current statistical concepts break new ground and raise questions which statisticians cannot afford to neglect and which call for further examination.

Failing to find a satisfactory basis on which to build a theory of economic rhythm out of the possibly rhythmic nature of some fundamental economic facts such as the regular recurrence of needs, Morgenstern turns in Part II of his book to the most convenient form of expression of economic movement, namely, the presentation of the fluctuation of different economic quantities in the form of time series, and inquires what they represent and whether we can infer any economic necessity for the apparent regularities they exhibit. He asks, in short, whether, if we have a sufficient number of series, we can build a forecast, on the basis of a knowledge of trend movements and of the amplitude and duration of any kind of cyclical movements about the trend. A trend, he points out, is merely the direction of movement of a time series over a long period of time, but since the economic order is one of interwoven quantities, processes, and courses, we cannot conceive of trends in isolation. At any given time, the value of an observation in a time series is at once determined by and determines the position of other quantities in the system, which themselves find ex-

[1] *Oskar Morgenstern, Wirtschaftsprognose: eine Untersuchung ihren Voraussetzungen und Moglichkeiten,* Vienna, 1928.

[2] It should be noted that Morgenstern uses the term "economic forecasting" in the sense of total forecasting (*Totalprognose*), that is to say, "the foreknowledge of the exact historical position of all those time series which are regarded as indexes of the general condition of the particular economic system concerned" (p. 35).

pression as one of the items in their appropriate trends. Trends as discovered by the statistician are of two kinds: they may be purely empirical, *i. e.*, obtained by moving averages or by the fitting of a free hand curve, or they may be expressed in mathematical laws. But when we turn to the phenomena of which the observed values constituting the series are but an expression, it is clear that the shape of the curve (the direction of the trend), and especially the changes of direction, the *breaks*, of the trend may be due either to the occurrence of events due almost entirely to natural causes, such as the discovery of further gold supplies or a bad harvest, or to causes which can be explained in view of what we know about the economic system. Even if this knowledge would justify us in accounting for such a degree of necessary regularity as would explain the expression of the trend in a mathematical law, it is clear that no such possibility attaches to the trends whose observations are conditioned by sporadic natural events. But if we admit, as we are forced to do, that all trends are interdependent, we are faced with the fact that regular trends, obeying a mathematical law, in part rest upon irregular trends which can never be expressed in such a form. As a result Morgenstern concludes that there are in fact no regular mathematically expressible trends, and any theory of conjuncture—which, as he points out, is necessarily implied in any attempt to forecast on the basis of our knowledge of the working of the economic system—building around the trend as a "normal" is to that extent building on sand.

Nor, he points out, is any better basis provided by the attempt to think in terms of a "general" situation whose movements can be described in terms of a general coefficient of progress (as in Cassel's work). The general coefficient can only be an average of the different trends, and as such has very little meaning, since there is no probability, but rather the reverse, that a given general coefficient of progress will be reflected in an equal coefficient applying to all the constituent trends, of which it is the average. If at any given moment we regard the situation as describable in terms of the combination of trend movements, we have to realize that this is a unique combination, since the constituent trends themselves do not remain the same for each arrangement of data. But if the combination is always unique, it furnishes a poor "normal" which can be used as a forecasting basis. On the one hand, a knowledge of the individual trends is insufficient; on the other, the measure of their interaction is so far conceivable merely as a statistical average whose apparent definiteness and exactness conceal the real differences in the situation and the relationships of which it should be the expression. It follows that if this normal disappears we can build but little upon the

statistically discovered regular deviations from it, *i. e.*, the cyclical movements of business-cycle theory.

But if we have thus to renounce forecasting on the basis of more or less graphical representations of the movement of those series which are taken to characterize the economy, can we not use the material which is summarized in the form of time series as a means to the discovery of relationships between different entities which may provide a basis on which we can build a prognosis? This question at once leads to the further one: *whether it is possible to summarize in the form of time series all those qualities of prices which have relevance for a complete price theory.* If this can be done, then by comparing a sufficient number of series we may learn something about their relative significance in the economic order. But when we consider the individual items of a time series, we see that any given quotation is what it is not merely because of other *contemporary* prices—it is in part influenced by the general price situation which has ruled in the past. Each equilibrium depends upon more than the immediately preceding one. The demand and supply situation indicated by a given price can be understood only if we know something of the prices which have ruled in previous periods and the contracts which have been made at those prices. On the one hand, every price is itself the result of a variety of previous prices: on the other hand, each price, numerically remaining the same through subsequent units of time, can be conceived of as having an influence extending into the future and governing future prices, although with ever-changing intensity. This quality Morgenstern calls the "time quality" of prices. Unfortunately it is not possible to determine once and for all what this influence is, either in relation to the number of other prices which will be affected or to the time which will elapse before the influence is finally worked out. Hence if we conceive of each price at any given time as influencing in some measure the future behavior of the economic system, similar price quotations may represent very different streams of influence, and we cannot tell at what stage of the potency of any given price we are obtaining our quotation.

It is of course undeniable that any event must, strictly speaking, reverberate throughout eternity. We may agree with Morgenstern that it would be rash to infer that two groups of identical price quotations mirror identical states of economic stresses and strains, stimuli and probable responses because of the possibility that the totality of events lying behind the one group differs from that lying behind the other, and because these events as well as the particular prices under consideration determine the reactions at the two instants of time to identical price situations. But the real question is whether Morgenstern's objection

to the use of price series as an expression of the group of influences within an economic system would lose its force if we were comparing not two groups of all price quotations at an instant of time but two series of all price quotations over a period of time—if we were dealing, in fact, not with a price photograph but with a price film. If we then discovered that the identity of all prices in the two groups extended over long periods of time, should we be safe in inferring that the response to the two identical price quotations under consideration would be identical? There is little doubt that for the purposes of practical forecasting we should be so justified, and Morgenstern's objection could only be made on the basis of fact, *i. e.*, that in practice we never should get such a series of identical quotations. So stated, his objection is one of some importance.

There is still the possibility that we may be able to use our knowledge of the movements of economic quantities as summarized in time series for the purposes of forecasting, by utilizing the concept of the Lag. If we have a group of time series all more or less interconnected, we may find that they do not all reach their high and low points simultaneously, but that some follow others. If the lag between series is apparently permanent, it may be possible to forecast the position of the later series when the movement of the earlier one is known. Ideally, of course the series whose movements are used as a basis for the forecast must represent some economic quantity of infinitesimally small economic significance; for it if be large, then there may be no use in the forecast, for the curve itself will partially represent those very movements of the economic system of which a forecast is needed. Thus, the pig iron production curve is of little use for this purpose, for the production of pig iron plays such an important part among the criteria by which general trade conditions are judged that by the time the pig iron curve has moved, the significant activity of which a forecast was needed will be actually taking place.

Morgenstern, however, suggests certain difficulties attending the use of lags as a basis of forecasting. Lags may be of three kinds, fixed, variable within limits, or dependent upon the existence of certain kinds of behavior or human rules and changing with changes in these. The permanent lags of the simple economy, which may reflect the technical production processes, etc., provide an example of the first kind. But these lags are but the results of the static nature of the economy and would disappear with that characteristic. Such lags can in the last resort rest only upon physical data. The second type of lag which may vary within limits may occur in relation to processes of production which may be extended or contracted, but whose alteration will

entail an alteration in costs. Under one set of circumstances, *e. g.*, high interest rates or general business prosperity, the entrepreneur may decide to spread the production over one period; under different circumstances he will retard or accelerate the period of time taken for production. The effect of this action will be to alter the lag between the quantity considered and others. The third kind of lag depends entirely upon human laws. The lag of wages behind prices depends *inter alia* upon the kind and duration of wage contract made: the relation between changes in bank deposits and the general business situation may be influenced by a change in the law governing the reserve which banks must keep. Morgenstern holds that such lags probably constitute the greater number of those which are used for forecasting. Unfortunately we have no means of knowing to which class any given lag belongs (in this Morgenstern seems to be unduly pessimistic), so that the value of using the lag as a basis of forecasting is much reduced.

Up to this point Morgenstern considers the use of statistics for economic forecasting which builds upon an assumed knowledge of the behavior of the economic world and of the nature and interrelationships of its parts. But even if we have to admit that our knowledge necessarily gives us no basis on which we can forecast, there is yet another approach to the problem which is opened up by the collection of economic data in statistical form. This is the possibility of calculating frequencies and applying probability theorems to our data. The bearing of the laws of probability on the fruitful use of statistical induction as a basis of forecasting seems to be twofold, and Morgenstern makes this section of his book less easy to read than it might be because of a failure to keep the two sets of considerations apart. On the one hand it may happen that the form into which the frequency theory of probability has to be cast is one which in no way assists the seeker after an economic forecast; on the other hand it may be held that the nature of the economic system is not such as to make tenable the essential assumptions without which any use of probability theories is unjustified.

If we are interested in the total forecast, no assistance can be obtained from any application of frequency theorems, for they can merely give information relating to the probability of occurrence of certain defined characteristics calculated on the basis of previously observed regularities of occurrence among a large number of cases. We can never be in a position to make a forecast about a unique individual event; we can merely estimate the chances that a certain characteristic will manifest itself in a given instance. We can calculate the chance that a man of forty has of not getting run over by an automobile: we

can never say exactly which man will meet with such an accident nor that a given man certainly will or will not. But in economic forecasting, Morgenstern claims, we wish to know exactly what will be the economic situation of a community at a future date. We desire specific information about the exact position or value of the quantity which we have chosen to represent our general situation. The forecast must be exact and correct and must relate to an individual case: approximate accuracy will not suffice.[1] And even if we relinquish the possibility of expressing the totality of an economic situation by an individual series, a "composite" or "general conditions" index, and conceive of it as the conjuncture of a large number of series, we have then to forecast the probability that a particular value or series of values will be attained, not by one series but by a great number. Even for a single series the probabilities are small: the final probability resulting from the multiplication of the separate probabilities is likely to be infinitesimal.

In order that a frequency constructed from a sample may be applied with a measurable degree of error to the totality of cases from which the sample is drawn, certain conditions must be fulfilled. Following Persons, Morgenstern points out that economic data fulfil none of these conditions and insists that it is useless to seek in statistical inference the key to economic forecasting that was denied by the approach of economic reasoning.

Lest there remain some optimists who refuse to despair, Morgenstern assails their position by adducing a third group of considerations. He denies, in Part III, the possibility of practical forecasting on a statistical or any other basis because of the fact that once a forecast has been made public, it will be acted upon by the members of the economic system, and this action will necessarily falsify the forecast in practice. The possible ways in which this knowledge may have this effect are analyzed in the last section of the second part of the book. Two comments may, however, be made to this "fundamental" objection to forecasting. In the first place, if it were possible on other grounds to apply probability theory to a sufficiently large number of cases in which the existence of a forecast was one of the factors in the economic system, then this possible reaction to the known forecast would be of no relevance, for it would itself be subject to the laws of probability. But in the second place, it would seem that this objection can scarcely be regarded as a *separate* objection to the possibility of total forecasting; it calls rather for discussion under the heading of our knowledge of the nature of the economic world. The question then resolves itself into

[1] This contention of Morgenstern's is challenged below.

deciding whether we have knowledge, not only of the general behavior of the members of the economy, but in particular of their relation to forecasts, and, if we did have this perfect knowledge, whether it would ever be possible so to frame a forecast that, taking into account the known reactions of the members of the economy, their joint result would be just such as to bring about the very forecast in question. This is a question which is indeterminate, but the problem is at least one which cannot be dissociated from the general problems relating to our knowledge of the nature of the economic society and of the interrelationships of its various parts.

The same reasoning applies to the final objection urged by Morgenstern, namely, that even if on our knowledge of the *economic* system forecasting were possible, the facts that men are guided by other than economic motives and that broad sociological events bear upon economic relationships present, in the absence of a perfected science of sociology, an insuperable barrier to forecasting. If the other formal requisites of probability theory were present, so that in entire ignorance of all interrelationships between sociological and economic elements it were yet possible to calculate the probability of the recurrence of a given combination expressible in purely economic form, then the fact of our ignorance of the nature of these sociological relationships would not invalidate the forecast. Indeed, our very lack of relevant knowledge might even support the probability, since there would be the greater certainty that the cases selected were chosen at random. Alternatively, we may regard this ignorance of the laws of sociology as an insuperable barrier to the construction of a reasoned system of theory which would suggest the essential relationships of the economic world. Here, again, the objection is in no sense separate from the two main points discussed in Part II of the book, but forms an integral part of that analysis. If we rely on probability theory it presents no difficulty in itself, provided the other formal requirements of this technique are present. If we build instead on our knowledge of the nature of the economic system, the lack of knowledge of the degree of influence of sociological factors is itself a proof that so far our theory of even the economic behavior of the economy (granted the economic reactions of these factors) is itself inadequate. In short, if we grant the validity of the arguments adduced in Part III, the elaborate discussion of the possibilities and assumptions of economic forecasting contained in Part II are superfluous if our interest lies solely in the total forecast.

Whatever the arrangement of the arguments, there is little doubt that Morgenstern has made out an effective case against the possibility of *total* economic forecasting. But it is possible to regret that he has

devoted so much time and attention to the establishment of so barren a conclusion. Even the most optimistic and ardent member of a conjuncture institute would scarcely assert a belief in the possibility of total forecasting as defined by Morgenstern. The practical and immediate problem is to obtain a better approximation to accuracy in our knowledge of the future than we have at present. Certainly from the point of view of the statistician with economic interests, the more important questions relate to the possibilities and probable effects of partial forecasts and to the relevance of the phenomena selected by the compilers of time series as the most significant from the point of view of the economic order. From this point of view, therefore, Morgenstern's book is useful and stimulating, not so much because of the main conclusion he establishes as because of the light he sheds upon current statistical concepts, of his analysis of the operation of the economic system, and of the refinements he makes in current price theory in the course of establishing his central thesis. As he points out, individual forecasts are an essential part of the economic system. All economic acts have to take the future into account, so that each is, in effect, a forecast. Unfortunately we have so far little knowledge of the criteria by which the members of the economy make their judgments, nor can we at present tell whether on the whole the separate judgments of individuals offset or intensify each other. Moreover, the individual forecasts are, in their way, forecasts of the total situation of the economy. We are dealing here, as Morgenstern points out, with one of the most unsatisfactory and least known and yet one of the most important parts of theory. It does, however, seem probable that the very existence of conjuncture institutes will enable us to generalize with more certainty about the nature of forecasts made by individuals. For there seems little doubt that the effect of such institutes will extend much farther than the mere correction and the rendering more precise of the data on which judgments were previously made. The very form in which the material is cast, and in particular the selection of the series which are presented to the business world as indicative of the economic situation, must inevitably work toward some degree of unification of the bases on which economic judgments are made. We cannot leave the problem, as Morgenstern does, by advising the institutes to confine themselves to the giving of information, for they cannot give information about *all* economic matters, and they must work on some theory of the more significant series, knowledge of whose movements will be of assistance to their clients. The probable effect of this concentration of the criteria on which judgments of the future economic situation will be based and the possible contrast with the previous heterogeneous

bases of judgments raise questions of immense significance upon which one might have wished Dr. Morgenstern to have touched. One of these relates to the choice of series to be reported. Have the devisers of economic services made a wise selection? What is the nature of the entity whose state all these series endeavor to mirror?

Furthermore, if we turn our attention from the total forecast and consider merely the forecasting of some of the symptoms of the working of the economy, it may well be asked whether all these incomplete forecasts are subject to the same objections that Morgenstern raises. One group he himself excludes. He conceives that it is possible to take all existing cycles and group them according to certain characteristics, *e. g.*, crises with and without panics, or crises which begin on the exchange or with the banks, and construct frequency tables which can be interpreted in the same way as the usual frequency tables, for all the logical requirements of the probability theory will be present. But this is all that a great many institutes do, and the reservation, therefore, is of such significance that some analysis of the use that might be made of this knowledge seems called for. In what ways will the actions of entrepreneurs and consumers be affected by the kind of knowledge that is amenable to such treatment? Will a knowledge of the probability of the occurrence of a given relation between the period of recession and the period of revival greatly affect the reactions of the active agents of the economy? We cannot answer this question without a theory of conjuncture upon which Morgenstern refrains from committing himself.

But not only do we wish to know much more about the nature of the symptoms to which the probability theory may be applied (for if these are many then the inability to make total forecasts becomes a matter of much less significance): we also need to know how many economic fluctuations are of the nature of seasonal or weekly movements, or are even due to certain relatively fixed institutional facts, such as religious or other feasts which can, however, be regarded as exogenous from the point of view of the economy concerned. The effect of these externally produced periodicities is to impress a certain relatively fixed pattern on the demand situation of the economy. Certain needs are known to be more intense at some times than at others. Whether or not the general trend of production is upward, people will buy more cream *at the same* price on Sunday than they will on Monday. Whatever the general level of trade, it will be more active just before Christmas than just after. The broadcasting of such general forecasts will undoubtedly tend to increase what Morgenstern, following Max Weber, terms the subjective rationality of the economy, and because the regularity of

the movements is due to causes which are beyond *economic* control (or, if we choose to put it that way, because we have now the rudiments of a classification of the recurrence of human wants), the foreknowledge of the event cannot be followed by action which will falsify the forecast. Milk suppliers may see to it that there is more cream on the market on Sunday than on Monday, but the fact still remains that there is a stronger *demand* for cream at any given price on Sunday than on Monday. The ultimate effect may be to regularize the flow of cream to the market and to prevent waste, but the truth of the original forecast will remain unaffected. The question then arises as to how many of the other fluctuations of economic activity are of this nature, that is to say, that can be traced to some of the external factors which the economic scientist can take as "given" or "ultimate" for the purposes of building up his theory of the operation of the economy and so can be made the basis of a forecast.

Morgenstern would argue that because of the interdependence of all the quantities in the economic system it is impossible to forecast the part without of necessity forecasting the whole. But if we give up at this point, we must abandon all hope of creating a usable economic science, or indeed any science at all. Interdependence is a fact we all recognize, and our real problem is to make judgments of proportion and relevance. Despite the general fact of interdependence, there are some quantities which are of more significance in the determination of one group of prices than are others. In attempting to forecast the price of pig iron for practical purposes, it is feasible to neglect the price movements of toy balloons or of English baronetcies, although we know that a complete analytic account of all the streams of influence affecting the price of pig iron would have to take them into account. Thus, even though we grant Morgenstern's contention that the interdependence of trends makes it impossible to conceive of permanently fixed trends amenable to expression in mathematical form, it may yet be the case that some kinds of movements can be expressed by trends of longer time validity than others. If we could discover some of the more stable of these trends, perhaps those whose relation to the rest of the economic system is relatively indirect in the sense that their values may be determined very largely by factors uninfluenced by economic considerations, we might conceive of some quantities as being relatively so much more significant in the economic order that even admitting the fact of interdependence within the system it would yet not be altogether illogical to regard them as relatively "causal" in their relation to the rest of the system. One such factor is the fact of population growth in the determination of which economic considerations may play a

relatively small part. If there are many such series, we may for prescribed periods be enabled to supply forecasts of some of the other economic relationships and series whose positions will be in this sense determined during the period of the validity of the longer and less economically susceptible trends.

Finally, there is a form of forecast which is admittedly incomplete but which may be made on the basis of a given theory of economic interrelationships. If our knowledge of economic cause and effect were greater, it might be possible to point to the logical consequences of a given form of economic activity, at least over a relatively short period of time. In fact, such warnings are by no means uncommon, and, indeed, their absence would be a strong indictment of the progress made by economics as a science, which would certainly have a poor record if it could not even now point to some of the more probable relationships of economic activities. To say that if such warnings are heeded the forecasted event will not occur is to make a statement of purely academic interest. The important question is the extent to which such statements of expected sequence are and could be made with advantage to the regular working of the economy.

It may be objected that the problems raised require so intense and detailed an analysis of the economic system that it is ungrateful, where so much has already been given, to ask for more. And yet one cannot but feel that Morgenstern's determination to prove the impossibility of the total prognosis has led him to disregard the analysis of the nature and significance of the different kinds of individual and partial prognoses that are in fact being made and which thus form a part of the system whose behavior is to be discussed. From the point of view of the construction of a satisfactory theory, it is important to know whether these forecasts can be classified and whether their existence is a factor tending to enhance our knowledge of the behavior of the undertakers and consumers, by increasing our knowledge of the stimuli to which they react. It is no answer to reply that the analysis of actual types of forecasts being made is a practical problem, with which the theorist who endeavors to trace the logical working of a system is unconcerned. The theory of the social sciences from which we derive our logical procedure can be constructed only from a knowledge of actual events. If we had a perfect system of theory, that which would be logically deduced from it would coincide with economic reality. It is clearly possible, of course, to make a certain number of postulates, and to build upon them a logical system. But as Simiand has pointed out, such a structure can be called an economic science only if the abstractions of the system correspond to the complex reality, if they "*respectent*

les articulations du reél, expriment quelquechose à la fois le distinct et le vrai par rapport à la multiplicité des cas individuels à laquelle elles (les abstractions) correspondent."[1] If we are to build up a satisfactory economic theory we must take into account all the relevant facts; we cannot stop when we have demonstrated the impossibility of a total forecast of the economic situation. The question that still remains to be answered after reading Morgenstern's book is how relevant are the actual forecasts that are being made at the present time. Does their existence necessitate any modifications in the system of theory which explained the operation of the economic world before statistical technique made possible the type of work and the incomplete forecasts which in greater or lesser degree are taken into account by consumers as well as entrepreneurs? To the answering of this fundamental question Morgenstern's demonstration of the impossibility of complete total forecasting helps us but little, but his penetrating analysis of the nature of certain statistical entities and of their ability to describe the totality of any given economic situation is a real contribution.

[1] *Statistique et Expérience*, p. 30.

relatively small part. If there are many such series, we may for prescribed periods be enabled to supply forecasts of some of the other economic relationships and series whose positions will be in this sense determined during the period of the validity of the longer and less economically susceptible trends.

Finally, there is a form of forecast which is admittedly incomplete but which may be made on the basis of a given theory of economic interrelationships. If our knowledge of economic cause and effect were greater, it might be possible to point to the logical consequences of a given form of economic activity, at least over a relatively short period of time. In fact, such warnings are by no means uncommon, and, indeed, their absence would be a strong indictment of the progress made by economics as a science, which would certainly have a poor record if it could not even now point to some of the more probable relationships of economic activities. To say that if such warnings are heeded the forecasted event will not occur is to make a statement of purely academic interest. The important question is the extent to which such statements of expected sequence are and could be made with advantage to the regular working of the economy.

It may be objected that the problems raised require so intense and detailed an analysis of the economic system that it is ungrateful, where so much has already been given, to ask for more. And yet one cannot but feel that Morgenstern's determination to prove the impossibility of the total prognosis has led him to disregard the analysis of the nature and significance of the different kinds of individual and partial prognoses that are in fact being made and which thus form a part of the system whose behavior is to be discussed. From the point of view of the construction of a satisfactory theory, it is important to know whether these forecasts can be classified and whether their existence is a factor tending to enhance our knowledge of the behavior of the undertakers and consumers, by increasing our knowledge of the stimuli to which they react. It is no answer to reply that the analysis of actual types of forecasts being made is a practical problem, with which the theorist who endeavors to trace the logical working of a system is unconcerned. The theory of the social sciences from which we derive our logical procedure can be constructed only from a knowledge of actual events. If we had a perfect system of theory, that which would be logically deduced from it would coincide with economic reality. It is clearly possible, of course, to make a certain number of postulates, and to build upon them a logical system. But as Simiand has pointed out, such a structure can be called an economic science only if the abstractions of the system correspond to the complex reality, if they "*respectent*

les articulations du reél, expriment quelquechose à la fois le distinct et le vrai par rapport à la multiplicité des cas individuels à laquelle elles (les abstractions) correspondent."[1] If we are to build up a satisfactory economic theory we must take into account all the relevant facts; we cannot stop when we have demonstrated the impossibility of a total forecast of the economic situation. The question that still remains to be answered after reading Morgenstern's book is how relevant are the actual forecasts that are being made at the present time. Does their existence necessitate any modifications in the system of theory which explained the operation of the economic world before statistical technique made possible the type of work and the incomplete forecasts which in greater or lesser degree are taken into account by consumers as well as entrepreneurs? To the answering of this fundamental question Morgenstern's demonstration of the impossibility of complete total forecasting helps us but little, but his penetrating analysis of the nature of certain statistical entities and of their ability to describe the totality of any given economic situation is a real contribution.

[1] *Statistique et Expérience*, p. 30.

NOTES

DOOLITTLE VERSUS THE KELLEY-SALISBURY ITERATION METHOD FOR COMPUTING MULTIPLE REGRESSION COEFFICIENTS

By Truman L. Kelley and Quinn McNemar, *Stanford University*

Following the publication by Kelley and Salisbury [1] of the iteration method for determining multiple correlation constants, there appeared an article by Tolley and Ezekiel [2] comparing the Doolittle method with the iteration method, in which they claimed the Doolittle method [3] to be the better. They computed the regression coefficients for the same six-variable problem as presented by Kelley and Salisbury, giving a comparison of the number of operations required by each method, and concluded that the Doolittle method required approximately one half the number of computations as the iteration method. Incidentally they have credited the iteration method with more arithmetic operations than actually exist for this problem. They further give the time required for the Doolittle but not for the iteration method. Nor do they take into consideration the simpleness of the majority of the operations involved in the Kelley-Salisbury method, many of which can be computed mentally in less than machine time.

Of still greater importance is the fact that the comparison made by Tolley and Ezekiel is for a six-variable problem, whereas it was stated by Kelley and Salisbury that the illustrative calculation involved six variables only to save space in presentation, and for no other reason. It was mentioned that the method had been found to be very serviceable in a sixteen-variable problem. Kelley and Salisbury did not compare their method with the Doolittle method, because they had not used that method, but with a method "based upon determinants." With no errors, computation by this method would be as short as by the Doolittle method. The method lacked, however, the adequate checking features of the Doolittle method and this was the cause of a

[1] Truman L. Kelley and Frank S. Salisbury, "An Iteration Method for Determining Multiple Correlation Coefficients," this Journal, 1926, Vol. XXI, pp. 282–292.

[2] H. R. Tolley and Mordecai Ezekiel, "The Doolittle Method for Solving Multiple Correlation Equations *versus* the Kelley-Salisbury Iteration Method," this Journal, 1927, Vol. XXII, pp. 497–500.

[3] For this method see F. C. Mills, *Statistical Methods Applied to Economics and Business*, 1924, pp. 491–98, 577–81.

substantial portion of the increased time credited to the determinantal method.

The present authors desire to express the conviction that the Doolittle method is a great improvement over the determinantal method used by Kelley and Salisbury, and they recommend it for a small number of variables—perhaps up to ten. There remains, however, the real issue in connection with which the iteration method was developed. Is it serviceable in handling a large number of variables?

It is the purpose of the present paper to give a summary comparison of the time involved and the number of calculations required by each method when applied to a twenty-one-variable problem, and to suggest some improvements in the iteration method which have been developed since its presentation in 1926. The computations were made by the authors, who used an electric Monroe calculator for the machine work.

TABLE I

REGRESSION WEIGHTS

Variable	Iteration method	Doolittle method
1	.083	.087
2	.081	.078
3	.166	.174
4	.304	.304
5	−.094	−.100
6	.049	.050
7	.018	.018
8	.094	.100
9	.035	.035
10	.026	.024
11	.074	.076
12	.036	.035
13	.071	.067
14	.268	.265
15	.056	.056
16	.022	.022
17	.058	.058
18	−.019	−.025
19	.045	.048
20	−.011	−.012
Multiple r =	.82781	.82783

The original data were scores on 20 psychological tests together with a criterion score measuring school success. The names of the variables are given in Table III and the intercorrelations in Table IV. The final β weights obtained may be of interest to psychologists and are given in Table I. As the tests themselves are not available to the public, the means and standard deviations are not given.

Table II gives the number of calculations required by each method for obtaining the multiple regression weights and the multiple coefficient of correlation for a twenty-one-variable problem.

TABLE II

Methods	Multiplications	Divisions	Additions	
			Totals	Items
Doolittle	2,548	20	307	2,968
Kelley-Salisbury	2,150	29	688	2,530

A glance at this table does not tell all that favors the iteration method. The Doolittle method required the recopying of 272 items as compared to none. Then it is to be remembered that one half the above "items added" in the iteration method can be accumulated in the machine, thus avoiding the double chance of errors involved in copying numbers from the dial and then replacing them in the machine for adding. The "back solution" and final substitution into the original equations are the only parts of the Doolittle method where this time-saving scheme is practical. This affects but one fifth the number of items added by the Doolittle method. It is to be further pointed out that one fourth or more of the multiplications in the Kelley-Salisbury method are so simple that they can be performed mentally, some 25 per cent being of this nature as the problem was worked by the authors. In fact, the method as applied by Salisbury[1] eliminates in the main process of the work all multiplications except those by 1, 2, 3, 4, and 5. It will be clear to one familiar with the method that the "hard" multiplications by 3 and 4 can also be eliminated, increasing the number of steps somewhat but decreasing the labor of each step. Trial by the computer will tell him how far it is economical to go in simplifying the work of each step. It is certainly entirely feasible (though this was not done by the present authors) to follow a procedure such that all multiplications involved in getting corrective factors are mental, being by the digits 1, 2, or 5. In the Kelley-Salisbury method three fourths of the totals obtained are made up of two items only, which likewise can be done more rapidly mentally than by machine. None of the Doolittle operations is so simple as to admit mental computation as better than machine calculation.

Since errors are likely to creep into computations, it is to be noted that the Doolittle method has the advantage of an absolute check at the end of each step; but for a twenty-one-variable problem this is not such a quick and ready check as claimed by Tolley and Ezekiel,

[1] F. S. Salisbury, "A Simplified Method of Computing Multiple Correlation Constants," *Journal of Educational Psychology*, January, 1929.

as is readily seen when it is pointed out that in nine of the twenty steps this check is not available until over 100 products are obtained and over 100 items added for each step. While the iteration method does not offer such an absolute method of checking except by recalculation of Table III (of article by Kelley and Salisbury), one's judgment ordinarily detects any significant error almost immediately after it is made. Nor does an error endanger further procedure, for with each Table III calculated (say once every 10 steps), a clean slate is presented.

TABLE III

DESCRIPTION OF TESTS*

0	Criterion	11	Form Combination
1	Arithmetical Reasoning	12	Computation
2	Written Directions	13	Vocabulary
3	Information	14	Sentence Completion
4	Synonym-Antonym	15	Disarranged Sentences
5	Practical Judgment	16	Logical Selection
6	Analogies	17	Copying Designs
7	Picture Completion	18	Pictorial Sequence
8	Series Completion	19	Pictorial Identities
9	Comparison Test	20	Recognitive Memory
10	Symbol-Digit		

* Correlations based upon data from Horace Mann School, Grades 5–7, total population 193.

As to the time element, the Doolittle method required 34 hours of computation and 28 hours and 35 minutes hunting for errors made. This includes 12 hours spent searching for an error in step 18 which never was found. While this unfound error affected the accuracy of the final Doolittle results, it did not prevent finishing the solution for present purposes. The same computer did the problem by the Kelley-Salisbury method in 19 hours and 20 minutes, including time spent finding errors. An expert or a professional computer could no doubt do both methods in somewhat less time.

The present work has been fruitful in that it has made apparent certain further short cuts for the Kelley-Salisbury method which heretofore have not been demonstrated. It has been found that the constants a, b, c, etc., (of the Kelley-Salisbury article) computed need not be more than four in number for a problem containing as many as twenty variables. This reduction in the number of a, b, c, etc., constants is accomplished by making successively delta corrections of opposite sign. If a certain correction made is $\delta_5 = .05$ then, in making the next correction, choose, of the largest indicated—say $\delta_{17} = -.03$, $\delta_8 = .05$, and $\delta_{12} = -.04$—the one of opposite sign, namely, $\delta_{12} = -.04$. This process may be continued for several steps, generally until another Table III (of the Kelley-Salisbury article) is calculated, thus doing away with the calculation of a constant of the a, b, c, etc., type.

TABLE IV

INTERCORRELATIONS

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	.4017																			
2	.5037	.3974																		
3	.6298	.3246	.4758																	
4	.6807	.2569	.4915	.6198																
5	.4745	.2680	.4740	.6509	.5453															
6	.4698	.4117	.3746	.3843	.3601	.3374														
7	.0946	−.0718	−.0284	.0559	.1028	−.0123	.0430													
8	.3468	.1398	.1565	.2464	.2423	.2379	.2493	.2147												
9	.2647	.0342	.1475	.3037	.2619	.2983	.1942	.1550	.3343											
10	.3553	.1064	.1774	.3400	.2989	.2284	.2245	.2773	.4424	.4940										
11	.1888	.2174	.0744	.1047	.0261	.2322	.3136	.1600	.1987	.0997	.2066									
12	.2272	.4433	.2173	.2015	.1590	.1566	.1382	−.0714	.1601	.0577	.0358	.0879								
13	.4982	.2968	.4181	.4831	.5180	.4807	.3615	.0116	.0862	.1651	.1230	.0379	.1042							
14	.6003	.2332	.3431	.4317	.4520	.3495	.3225	.0000	.1727	.0720	.2139	.0463	.1090	.3575						
15	.5448	.2095	.3611	.5433	.5554	.5460	.3604	.0152	.2239	.2253	.1569	.0937	.2002	.4907	.5049					
16	.2379	.1968	.1698	.1820	.2628	.1304	.3097	.0793	−.0139	−.0627	.0033	.1122	.0910	.2126	.1747	.1820				
17	.1161	−.0190	−.0347	.0848	.0622	.0125	.0147	.0106	.1676	.1218	.2641	.1758	−.1930	.0113	.0101	−.0962	−.1443			
18	.1795	.0618	−.0147	.1347	.1692	.1142	.1498	.3391	.3436	.0995	.2771	.2784	−.0149	−.0239	.1343	.0856	.1113	.1904		
19	.2906	.2665	.2490	.1688	.1207	.0332	.2943	.0886	.1585	.0511	.1414	.2114	.0201	.2177	.2231	.1423	.0665	.0819	.1477	
20	.2142	.1053	.2794	.2183	.1645	.2140	.1928	.2144	.2610	.3296	.3019	.1267	.1464	.1226	.1423	.2077	−.0102	−.0410	.1649	.0058

Another point to note is that of a number of indicated corrections of about the same size one should always make that one first that pertains to the variable already having the larger weight. By the exercise of a little judgment, the number of corrective factors, designated by the small Greek letter delta in the Kelley-Salisbury article, can be reduced greatly. Surely a clerk should be able to handle computation involving such elementary exercise of judgment. At any time Table III can be calculated as a check to previous work, and doing so is at times helpful for further computation. It needs be mentioned again that this table does not require much time, since the addends can be allowed to accumulate in the machine.

It is believed that for a problem containing a small number of variables the Doolittle method is the best available, but it seems evident that for a large number of variables the iteration method is much the shorter. If great accuracy is demanded the Doolittle method tends to become the more expeditious, but if computation accuracy of the order of .1 of the probable error of the regression coefficient involved (assuming samples of less than 20,000) is sufficient, and the authors do so deem it to be, then for practical purposes there is no real point in choosing the longer Doolittle method.

It is interesting to note that the time for the solution of a twenty-one-variable problem, whether the 20 hours of the iteration method or the 60 hours of the Doolittle method, is so short as to be quite negligible in comparison with the other time costs of a twenty-one-variable problem.

The long delay in presenting this present reply to Tolley and Ezekiel's 1927 criticism of the iteration method is due to the fact that the present authors have felt it incumbent upon them actually to test out the two methods in question upon a critical problem before rushing into print, and they have not earlier had the time in which to do this.

FORMULAS FOR PROBABLE ERRORS OF COEFFICIENTS OF CORRELATION

By Walter Crosby Eells, *Stanford University*

There are three formulas for the probable error of a coefficient of correlation concerning the proper form and use of which there has been some confusion. Two of them are in a form that is somewhat awkward and laborious for common use. The object of this paper is to point out the complete and correct form of these formulas and to tabulate their values in a form suitable for convenient use.

The first is the well-known formula for the probable error of the correlation coefficient, r, when it is found by the Pearson product-moment method. It is

$$\text{P.E.}\, r_{\text{Product-Moment}} = \frac{0.6745(1-r^2)}{\sqrt{N}} \quad \text{(A)}$$

The second is applicable when the probable error is desired for *rho* (ρ), the coefficient of correlation found by the Spearman method of squared differences in rank. As given by Pearson [1] it is

$$\text{P. E.}_\rho = \frac{0.6745(1-\rho^2)}{\sqrt{N}}\left\{1+.086\rho^2+.013\rho^4+.002\rho^6\right\} \quad \text{(B)}$$

The third is applicable when the Pearson coefficient, r, is found from ρ by means of the transmutation formula $r=2\sin\left(\frac{\pi\rho}{6}\right)$. As given by Pearson [2] it is

$$\text{P. E.}_{r_{\text{found from }\rho}} = \frac{0.7063\ (1-r^2)}{\sqrt{N}}\left\{1+.042r^2+.008r^4+.002r^6\right\} \quad \text{(C)}$$

Of some twenty well-known books on statistics and statistical methods—educational, economic, biological, and mathematical—only one was found in which these last two formulas (B) and (C) were correctly and completely given. Yet if ρ is to be used at all, and if r is to be estimated from it by the sine relationship, it seems obvious that the P. E. of both values should be correctly and conveniently available as an aid in determining their significance and limitations.

[1] Karl Pearson, "On Further Methods of Determining Correlation," Drapers Company Research Memoirs, Biometric Series IV. Mathematical Contributions to the Theory of Evolution, XVI. London, 1907, p. 30, formula xlix, and p. 31. Pearson gives 0.67449 instead of 0.6745.

[2] Pearson, *loc. cit.*, p. 31, formula (1). Pearson gives 0.70633 instead of 0.7063.

Both formulas (B) and (C) are correctly and fully given as above by Rietz and Crathorne in the National Research Council's *Handbook of Mathematical Statistics*,[1] and their use is there correctly stated.

Kelley [2] states that "the probable error of ρ as determined by Pearson (1907 further) is $\frac{.7063(1-\rho^2)}{\sqrt{N}}$, or approximately 5 per cent greater than the probable error of r." He gives no formula for P. E. of r when found from ρ. The coefficient 0.7063 is the correct one for P. E. of r when found from ρ, not for ρ itself, according to Pearson's original memoir. The expression in parentheses, composed of the higher powers, needs to be added for completeness. When this is done it is found, as Pearson states,[3] that "the maximum difference, as r approaches unity, is 10 per cent." The larger coefficient accounts for about 5 per cent, the terms in the parentheses, for the other 5 per cent.

Holzinger [4] makes the same mistake in his new textbook, giving the P. E. of ρ as $\frac{.7063(1-\rho^2)}{\sqrt{N}}$, and not mentioning the P. E. of r found from ρ. He gives no reference for the derivation of this formula.

Chaddock,[5] in discussing Spearman's rank method of correlation, does not give the formula for its probable error, but states that "the probable error of ρ, as determined by Pearson, is about 5 per cent greater than the probable error of r."

West [6] gives the probable error of the "rank coefficient" as $\frac{0.6745}{\sqrt{N}}(1-\rho)$ and credits it to Pearson.

Garrett [7] gives no value of the probable error of ρ, but he gives for the probable error of r when found from ρ, $\frac{.7063(1-r^2)}{\sqrt{N}}$, and states that "the P. E. of an r found from a ρ, is about 5 per cent larger than the P. E. of the product-moment r."

Hines [8] gives $\frac{0.706(1-\rho^2)}{\sqrt{N}}$ for the probable error of ρ, no value for P. E. of r found from ρ.

[1] H. L. Rietz (Editor), *Handbook of Mathematical Statistics*, Boston, 1924, p. 132, formulas 28 and 30. It is worth mentioning in passing, however, that the formula for transmutation of ρ into r is there given incorrectly as $r=\sin\left(\frac{\pi\rho}{6}\right)$ (formula 29) instead of $r=2\sin\left(\frac{\pi\rho}{6}\right)$.

[2] T. L. Kelley, *Statistical Method*, New York, 1923, p. 194. [3] Pearson, *op. cit.*, p. 31.

[4] K. J. Holzinger, *Statistical Methods for Students in Education*, Boston, 1928, p. 280, formula 133, and p. 355.

[5] R. E. Chaddock, *Principles and Methods of Statistics*, Boston, 1925, p. 301.

[6] C. J. West, *Introduction to Mathematical Statistics*, Columbus, 1920, p. 92. Slight change of notation used, for uniformity. Note ρ for ρ^2.

[7] H. E. Garrett, *Statistics in Psychology and Education*, New York, 1926, p. 191.

[8] H. C. Hines, *A Guide to Educational Measurements*, Boston, 1923, p. 33.

Brown and Thomson [1] give no value for P. E. ρ, but for the probable error of r when found from ρ they give $\frac{.7063\ (1-r^2)}{\sqrt{N}}$.

Many other authors, although they treat the method of correlation by squared rank differences, do not mention its probable error in connection with it.

Thus, all of the books quoted, except the *Handbook of Mathematical Statistics*, are incorrect or incomplete in their formulas for the P. E. of ρ, and of r when found from ρ, and in their statements regarding its percentage difference from the value of the P. E. for the product-moment r. The derivation of the formulas in Pearson's original memoir is, in his own words, "admittedly lengthy, but [he has] not seen [his] way to shorten it, and the main point is to reach by some road the expression for the probable error." [2] It begins on page 25, and runs through page 31. With reference to both formulas (B) and (C), he points out (page 31) that in each case the probable error is in excess of that given by the product moment method by a maximum amount of 10 per cent, this excess decreasing as the correlation coefficient approaches zero.

Since the formulas (B) and (C) are so rarely found in convenient form and are so tedious to calculate, it has seemed desirable to compute their values and to tabulate them for the entire range of the correlation coefficient. The following table gives the probable error from each of the three formulas (A), (B), and (C), to three significant figures, for values of the correlation coefficient varying by intervals of 0.01. The entries in the table are all for $N=1$. The value of the P. E. for $r_{\text{Product-Moment}}$, ρ, or $r_{\text{found from }\rho}$ may easily be found for any population N of correlated measures by taking the proper entry from the table and dividing it by $\sqrt{N}$.

No effort is made here to consider the limitations on N for small populations leading to the desirability of using $\sqrt{N-1}$ or some other function of N. A good discussion of the limitations of probable error formulas from this standpoint may be found in Fisher's recent book.[3] If it is desired to use $\sqrt{N-1}$ for small populations, the table is equally useful.

The computations on which the table is based were carried to five places and checked by differences.

[1] William Brown and G. H. Thomson, *The Essentials of Mental Measurement*, Cambridge, 1921, p. 103.
[2] Pearson, *op. cit.*, p. 25.
[3] R. A. Fisher, *Statistical Methods for Research Workers*, Edinburgh, 1925, chapter VI.

TABLE FOR CALCULATING PROBABLE ERROR OF CORRELATION COEFFICIENT WHEN FOUND BY PEARSON PRODUCT-MOMENT METHOD (A), SPEARMAN (ρ) SQUARED RANK DIFFERENCE METHOD (B), OR BY TRANSMUTATION OF ρ INTO r (C)

(Computed for $N=1$. To find P. E. by Pearson's formulas for any population, divide tabular entry by $\sqrt{N}$)

Value of correlation coefficient	P.E.$_{r_{\text{Product-Moment}}}$ (A)	P.E.$_\rho$ (B)	P.E.$_{r_{\text{found from }\rho}}$ (C)	Value of correlation coefficient	P.E.$_{r_{\text{Product-Moment}}}$ (A)	P.E.$_\rho$ (B)	P.E.$_{r_{\text{found from }\rho}}$ (C)
.00	.6745	.6745	.7063	.50	.506	.517	.536
.01	.674	.674	.706	.51	.499	.511	.529
.02	.674	.674	.706	.52	.492	.504	.521
.03	.674	.674	.706	.53	.485+	.497	.514
.04	.673	.674	.705+	.54	.478	.490	.507
.05	.673	.673	.705−	.55	.470	.483	.499
.06	.672	.672	.704	.56	.463	.476	.492
.07	.671	.671	.703	.57	.455+	.469	.484
.08	.670	.671	.702	.58	.448	.461	.476
.09	.669	.670	.701	.59	.440	.454	.468
.10	.668	.668	.700	.60	.432	.446	.459
.11	.666	.667	.698	.61	.424	.438	.451
.12	.665−	.666	.697	.62	.415+	.430	.442
.13	.663	.664	.695−	.63	.407	.422	.434
.14	.661	.662	.693	.64	.398	.413	.425−
.15	.659	.661	.691	.65	.390	.405−	.416
.16	.657	.659	.689	.66	.381	.396	.407
.17	.655+	.657	.687	.67	.372	.387	.397
.18	.653	.654	.684	.68	.363	.378	.388
.19	.650+	.652	.682	.69	.353	.369	.378
.20	.648	.650−	.679	.70	.344	.360	.368
.21	.645−	.647	.676	.71	.334	.350+	.358
.22	.642	.645−	.673	.72	.325−	.341	.348
.23	.639	.642	.670	.73	.315+	.331	.338
.24	.636	.639	.667	.74	.305+	.321	.328
.25	.632	.636	.664	.75	.295+	.311	.317
.26	.629	.633	.660	.76	.285−	.300	.306
.27	.625+	.629	.657	.77	.275−	.290	.296
.28	.622	.626	.653	.78	.264	.279	.285−
.29	.618	.622	.649	.79	.254	.269	.273
.30	.614	.619	.645+	.80	.243	.258	.262
.31	.610	.615−	.641	.81	.232	.246	.251
.32	.605+	.611	.637	.82	.221	.235+	.239
.33	.601	.607	.632	.83	.210	.224	.227
.34	.597	.603	.628	.84	.199	.212	.215+
.35	.592	.598	.623	.85	.187	.200	.203
.36	.587	.594	.618	.86	.176	.188	.191
.37	.582	.589	.613	.87	.164	.176	.178
.38	.577	.584	.608	.88	.152	.164	.165+
.39	.572	.580	.603	.89	.140	.151	.153
.40	.567	.575−	.597	.90	.128	.138	.140
.41	.561	.569	.592	.91	.116	.125+	.126
.42	.556	.564	.586	.92	.104	.112	.113
.43	.550−	.559	.580	.93	.091	.099	.100
.44	.544	.553	.574	.94	.079	.085+	.086
.45	.538	.548	.568	.95	.066	.072	.072
.46	.532	.542	.562	.96	.053	.058	.058
.47	.526	.536	.556	.97	.040	.044	.044
.48	.519	.530	.549	.98	.027	.029	.029
.49	.513	.524	.542	.99	.013	.015−	.015−
				1.00	.000	.000	.000

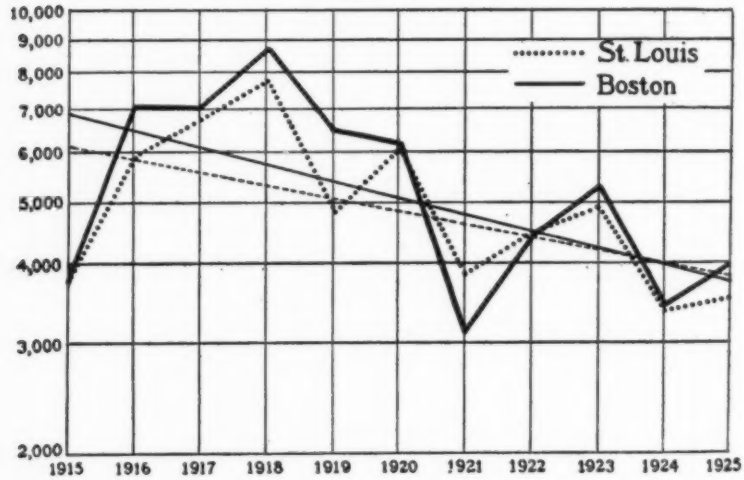
A STATISTICAL ANALYSIS OF JUVENILE EMPLOYMENT IN ST. LOUIS

By RALPH and MILDRED FLETCHER, *Washington University*

In a recent study, *Juvenile Employment and Labor Mobility in the Business Cycle*,[1] Dr. Maurice B. Hexter has made use of employment permits issued to minors as an index of child employment. After subjecting this data to statistical analysis he discovers some very interesting relationships to exist between the cyclical fluctuations of this measure of juvenile employment and accepted indexes of business conditions. That such a study merits a continuation with the data

CHART I

TRENDS



available in other cities is apparent. The purpose of this article is to compare the characteristics of employment permits issued in St. Louis with those issued in Boston and to discover to what extent the two may be affected by similar forces.

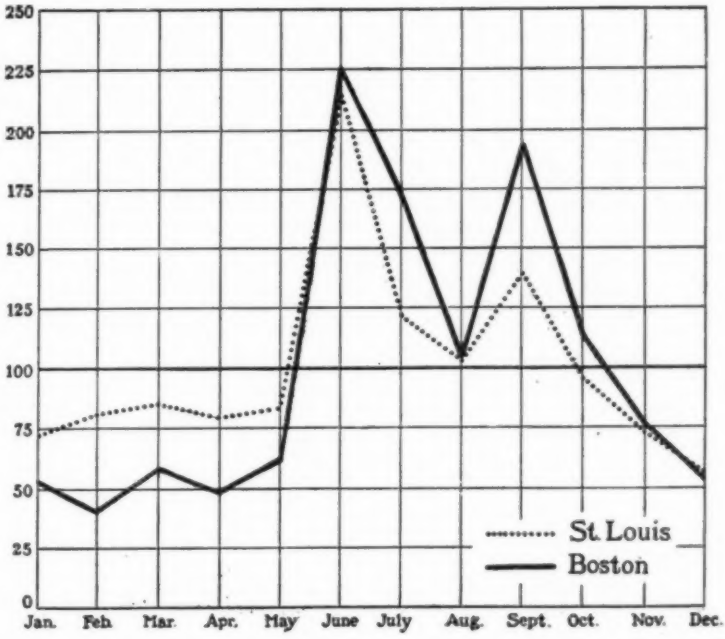
The St. Louis data consist of the total work permits issued to children under Section 5 of the Child Labor Laws of Missouri, which reads that "no child under the age of fourteen years shall be employed at any gainful occupation when the school of the district in which the child resides is not in session, unless such child shall have a permit certificate issued in the manner and by the authority herein directed, and no

[1] Child Labor Committee, Boston, Massachusetts, 1927.

child over fourteen years of age and under sixteen years of age shall be employed at any gainful occupation while the public school of its district is in session, unless such child has a permit in the manner and by the authority herein provided for, and no permit certificate shall be in force without renewal for a period longer than three months from the date of issuance thereof." The provisions of the act do not apply to children engaged in agriculture and domestic service. The data for

CHART II

SEASONAL INDEXES



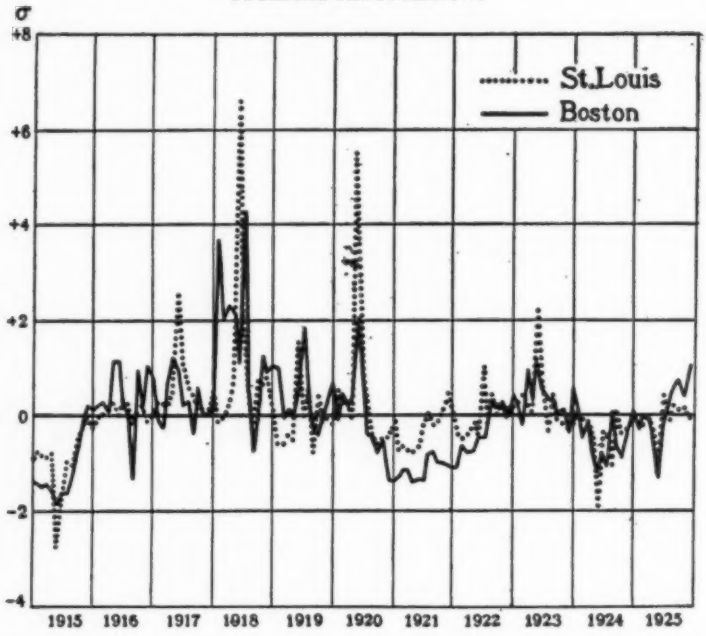
work permits in St. Louis are contained in the annual reports of the St. Louis school board and are available from January, 1913, to May, 1927. For purposes of comparison, only the period studied by Dr. Hexter is used. Series No. 1 of Dr. Hexter's study (New Employment Certificates) seems to furnish the best basis for comparing St. Louis with Boston.

Chart I is a comparison of the trend of St. Louis work permits with the trend of new employment certificates issued in Boston. The formula for the St. Louis trend is $\log Y = 2.6o638 - .o19o7X$. The agreement of the two series in this regard is quite noticeable. Chart II is a comparison of the seasonal indexes which are both constructed

by the Personian median link relative method. Except for the fact that the issuance of St. Louis permits continues more steadily throughout the early months of the year, the seasonal variations are substan-

CHART III

CYCLICAL FLUCTUATIONS



tially the same. The seasonal values for St. Louis are as follows: January, 72; February, 80; March, 85; April, 79; May, 83; June, 214; July, 121; August, 103; September, 139; October, 96; November, 73; December, 56.

WORK PERMITS IN ST. LOUIS

(Standard units)

	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925
Jan.	− .92	− .26	+ .01	+ .50	+ .29	− .19	− .12	+ .09	+ .19	− .15	− .03
Feb.	− .78	− .06	+ .28	− .10	− .60	+ .54	− .72	− .38	+ .29	+ .03	− .14
March	− .86	+ .01	+ .25	− .01	− .59	+ .27	− .62	− .49	+ .46	− .05	− .15
April	− .87	+ .19	+ .26	+ .23	− .39	+ .28	− .72	− .38	+ .21	− .08	− .02
May	− .78	+ .23	+ .50	+ .70	− .50	− .01	− .73	− .12	+ .09	− .44	− .29
June	−2.71	0	+2.53	+6.61	+1.57	+5.53	− .64	− .37	+2.21	−1.89	−1.14
July	−1.73	+ .33	+1.09	+1.97	0	+1.01	− .17	+1.04	+ .66	− .34	+ .50
Aug.	− .97	+ .26	+ .60	+ .58	+ .55	+ .35	+ .05	− .17	− .30	− .36	− .06
Sept.	−1.03	− .15	+ .42	− .28	− .76	− .39	− .19	+ .49	+ .43	−1.03	+ .26
Oct.	− .49	+ .06	+ .16	+ .77	+ .43	− .54	− .10	+ .04	− .02	+ .07	+ .13
Nov.	− .27	+ .12	+ .12	+ .68	− .10	− .46	+ .17	+ .28	− .16	− .34	+ .19
Dec.	− .05	− .11	+ .16	+ .85	− .03	− .45	+ .50	+ .13	− .29	− .26	− .01

Chart III gives a comparison of the cyclical fluctuations expressed in standard deviation units. The correlation between these two series is as follows: St. Louis leading Boston by one month +.571, synchronous +.521, St. Louis lagging behind Boston by one month +.455.

The marked character of agreement between the various statistical measures of these two series, so widely separated geographically, increases the significance of the granting of work permits as a reliable measure of the fluctuation in the juvenile labor market.

CENTRAL COLLECTION OF SOCIAL STATISTICS

A dinner meeting of the American Statistical Association was held at the Aldine Club, 200 Fifth Avenue, New York City, on Tuesday evening, January 29, 1929. Seventy-nine persons were present.

The first speaker of the evening was Dr. Godias J. Drolet, statistician of the New York Tuberculosis and Health Association. He spoke on "Central Statistical Services in Health Work in New York City." Dr. Drolet stated that the first system of central reporting in this field is the one developed by the members of the Association of Tuberculosis Clinics of the City of New York. The second includes the reports of the members of the Tuberculosis Sanatorium Conference of Metropolitan New York. The third system has been developed by the Cardiac Clinics coöperating with the Heart Committee of the New York Tuberculosis and Health Association. The fourth, closely allied to the previous group, includes the reports of Convalescent Homes for Cardiac Children. The fifth is the central report system of the Bellevue-Yorkville Health Demonstration.

The first four types of report systems refer to work that has been highly specialized and standardized (particularly in the case of tuberculosis) for a number of years, and to that extent have, therefore, become quite elaborate. The fifth system, namely, the reports exchanged at the Bellevue-Yorkville Health Demonstration, refers, on the other hand, to extremely varied public health and welfare work in a section of midtown New York.

The Association of Tuberculosis Clinics first began requiring its members to furnish a monthly standard statement of their work some twenty years ago. Up to 1918 these monthly reports were sent to the Clinic Association for official use only, and were not exchanged between members. In 1919 Dr. Drolet consolidated the individual reports into a single statement and began reproducing copies by the zincograph process and circulating them to all the members of the Association, so that the individual members might have an opportunity to see what was going on in every district of New York. At present thirty clinics report upon their work. The reports are tabulated individually and by boroughs. All the clerical work incident to the central tabulation is done at the headquarters of the Association.

The report for each clinic gives the case load at the beginning of the observation, the admissions, the discharges, and the number of those remaining under observation, with subdivision as to whether they were tuberculous, non-tuberculous, or suspects only. Data for children are separated from those pertaining to adults. Six items on the report describe the medical condition of the permanent cases of tuberculosis, so that one may get an idea of the results of treatment at each place. Two more items describe the positive history of the patients. Six more items relate to the nurses' visiting work. Home supervision of tuberculosis is a large factor with these organizations. Separate records are kept for clinic and non-clinic cases. More than 120,000 visits by nurses are made annually in this field alone, and it is important to see what effect these visits have on tuberculosis.

Seven items are devoted to recording the official, complete tuberculosis registration in every district and in the city as a whole. At present there are some 26,000 tuberculosis cases registered in the entire city. The data are so subdivided that one can ascertain whether the patients are under clinic care, are at home, are under care of private physicians, are in hospitals in the city, are in sanatoria, etc.

The reports also contain information as to the frequency of the clinics, the sum of the hours of their service, and the personnel, physicians, and nurses available. They account for the examination of almost 40,000 persons annually.

The report system of the Association of Tuberculosis Clinics served during the war as a model for the organization of similar work in France. At present, a report system, including 400 dispensaries or health centers, and covering almost every department in France, has been built up successfully and maintains a constant interchange of reports. The New York system of central report statements has been followed recently by tuberculosis clinics in the city of Philadelphia. It has also been utilized by a second group of associations in New York, which will be referred to later—the Cardiac Clinics.

The second system of central statistical service, developed in New York City more recently, is that of the Tuberculosis Sanatorium Conference. The sanatorium reports are not so elaborate as those of the clinic group. They do, however, furnish essential data for twenty institutions, sixteen of which are hospitals and sanatoria and six special institutions. They have a bed capacity or places for 4,439 patients and care for more than 13,000 consumptives annually. The primary purpose of these local sanatorium reports is to observe the trend of the volume of tuberculosis hospital work in the Metropolitan Area; for sanatorium treatment is held to be the keystone in the control of tuberculosis.

The third group of central report systems in New York City is that of the Cardiac Clinics, organized in coöperation with the Heart Committee of the New York Tuberculosis and Health Association.

The first four types of central statistical services described were devoted to specialized and standardized single lines of health activities. The fifth system, on the other hand, approaches practically every variety of public health work done by official or private agencies in one of the busiest sections of New York City, namely, the Bellevue-Yorkville district on the East Side, which has been the seat of the Bellevue-Yorkville Health Demonstration, one of the three New York health demonstrations inaugurated by the Milbank Memorial Fund. Beginning in 1927, the Statistical Division of the Demonstration attempted to devise an elementary form of gathering monthly reports from the various organizations that worked there. This was difficult because of the great variety of services and their dissimilarity. The present report form was devised in order to give the Demonstration officials some measure of what was going on. It has three broad divisions, the first items relating to work at the Demonstration itself, the second to field work in addition, and the third to the personnel involved. The reports are simply an attempt to measure the volume of personal contacts that were being established between residents of that district and the health and welfare workers. In 1927 more than 100,000 such contacts were made.

Measurement of field work has proved difficult. The simplest measuring unit is the count of so-called "home visits." It has been necessary to add also an item for "case visits." There is need for closer definition of the unit in public health and welfare visiting work. At present a "home visit" may mean merely the number of doors looked at by the workers, whether the cases were actually seen or not. Legitimately, no visits should be counted when the person concerned has not been seen.

In regard to personnel, it is necessary to take account of whether the workers are engaged full time or part time.

The central reporting systems cost relatively little to operate and, while they cannot take the place of intensive, special surveys, they have advanced constructive work.

The second speaker of the evening was Dr. Ralph G. Hurlin, statistician of the Russell Sage Foundation. His topic was "Central Collection of Welfare Statistics and the Outlook for an Index of Dependency." He called attention to the fact that twelve years ago Dr. Rubinow established a bureau of social statistics in New York City and made the first attempt to record changes in dependency by means of a statistical index. Funds to support this bureau soon gave out, however, and the work was necessarily discontinued.

Summarizing the present status of the collection of welfare statistics, Dr. Hurlin enumerated a series of recent projects. In 1926 Raymond Clapp published a report giving comparative data on volume and cost of welfare work for the year 1924 in nineteen cities. From this beginning there developed last year, at the initiative of the Association of Community Chests and Councils, a central registration scheme covering twenty-four fields of welfare activity in twenty-nine cities. This work is being done at the University of Chicago, under the direction of Mr. A. W. McMillen.

In Pennsylvania several steps have been taken in the direction of better welfare statistics. The Public Charities Association of Pennsylvania is conducting a study in which several experiments in central reporting are being made. Analysis is being made in this study of trends and cycles in poor relief, as shown by a fifty-year series of welfare statistics for the counties of Pennsylvania compiled at the Pennsylvania Department of Welfare by Emil Frankel. The State Department is also instituting a system of central reporting, covering records of admission and discharge of inmates of almshouses and other institutions in the state.

The Welfare Council of New York is devoting much attention to current statistics. For several fields of welfare work, regular reporting is already in operation, and reporting in other fields is being organized. Dr. Hurlin presented a chart, prepared by Miss Maude E. Stearns of the Council, showing close inverse correlation between factory employment and the volume of services rendered by the New York City Municipal Lodging House.

Reference was made to the work of the Committee of the American Statistical Association on Institutional Statistics, under the chairmanship of Dr. Horatio M. Pollock, in standardizing statistics of institutions, and to the plans the committee now has for introducing a system of standardized statistics for state welfare departments. That good progress has been made by professional organizations in

several fields of social work in standardizing terminology and definitions for statistics was also pointed out.

The collection of family welfare and relief statistics by the Russell Sage Foundation was begun in January, 1926, and since then monthly statistics have been collected and analyzed currently. Similar experimental collection of statistics has also been carried on in three other fields for the joint purpose of improving the periodic data and testing their value as dependency series.

The discussion of the two preceding papers was begun by Dr. I. M. Rubinow. He expressed the opinion that it was unfortunate that dependency indexes were not appreciated in 1913 as they are at present. In those days the foundations thought that such figures were not amenable to statistical treatment and that efforts along this line represented social frills. The result is that we do not know what changes have taken place during the period 1913–1926. Dr. Rubinow questioned whether family agencies represent fairly the status of relief work in the United States. Family agencies are becoming so sociological that they do little relief work. The probabilities are that municipal lodging houses offer a better gauge of dependency. As a matter of fact, our indexes ought to measure *economic need* rather than the volume of relief actually given. This distinction is important because the exhaustion of funds in the treasuries of the relief organizations may pull down the extent of relief given, even though the *need* is as great as ever.

The discussion was continued by Miss Jessamine S. Whitney. She stressed the importance of the interpretation of the statistical tables which Dr. Drolet had spoken of as accompanying the tables sent out to the various clinics. Miss Whitney felt that tables, if not interpreted, were practically useless to the public. She also called attention to the fact that the oldest central recording system in the United States is that of the registration of deaths by the Bureau of the Census, this registration having been begun in 1900.

Dr. F. Stuart Chapin stated that he found interest in social statistics far more intense in the United States than in Europe. He also wished to emphasize the fact that it is much more feasible to measure relative changes in social phenomena than it is to measure absolute quantities.

Dr. Joseph Gillman called attention to a study made in Cleveland which showed close correlation between the number of families seeking relief and the standard indexes of business activities.

Dr. Horatio M. Pollock stressed the fact that an upward trend in the volume of relief given is likely to indicate, not an increase in need, but rather an increase in resources of relief agencies. He pointed out, as an example, the fact that insanity appears, in general, to be most prevalent in the states in which the best facilities are furnished for the care of the insane.

WILLFORD I. KING

ECONOMIC OUTLOOK FOR 1929

On Thursday evening, March 14, another of the dinner meetings of the American Statistical Association was held at the Aldine Club. Three hundred and one persons were present. Mr. Carl Snyder, former president of the Association, presided during the early part of the evening, but later his place was taken by Dr. Warren M. Persons, also a former president of the Association. The general topic discussed by the speakers of the evening was the "Economic Outlook for the Remainder of 1929."

The first speaker of the evening was Mr. Paul Clay, of the United States Shares Corporation, who spoke on the subject, "The Financial Outlook." In Mr. Clay's view, the demand for stocks varies directly with the surplus cash which the people of the country have after they have paid all living and business expenses and the cost of ordinary construction and improvements. The stock market has been high recently because this surplus has been high, and it in turn has been high because the income of the people has been large. Because of the increased national income, we are justified in expecting that the volume of brokers' loans will permanently be larger than it has been in the past. It appears, however, that stock speculation has of late increased the volume of such loans to an abnormal height, for recently this volume has been *in toto* 11½ per cent as great as the aggregate bank deposits. At the peak of previous bull markets, it has never exceeded 4½ per cent of the aggregate of bank deposits.

In general, interest rates and stock prices are tied together. Stock buying raises interest rates, and these rates do not fall much until the bull movement in stocks is past. As long as stocks are rising, easy profits induce people to buy more stocks rather than to pay off loans; hence, as long as the stock market continues to rise, it tends to absorb all available funds, without regard to how extensive these available funds may be.

The remark is frequently heard now that this is a *permanent* bull market. This view is far from being new, having been held in most past bull markets. The boom in stocks will last just as long as optimism and confidence continue, for it is purely psychological in nature.

Much attention is being devoted to the fact that current yields on stocks are at present extremely low. As a matter of fact, they are not much lower than was the case in 1902 and 1906. The truth is, however, that the ability to borrow and buy, and not the rate of return, is primarily responsible for the bull market. To sustain a bull market, three things are necessary—courage, capital, and credit. The first of these three to fail is, ordinarily, courage. As soon as confidence disappears, the break will come. At present there is strong evidence that confidence is weakening. Bankers are showing a disposition to curtail credits. The large investors have apparently been disposing of their stocks since the first of February.

The bull market cannot continue without the continual pouring in of new money; therefore active industry is essential to the maintenance of a bull market. It is frequently contended that brokers' loans finance industry and assure its

continuance. This is largely fiction, for an analysis of the facts indicates that brokers' loans do not, at the outside, finance industry by more than 8 per cent of the total amount of these loans.

The fact that bond prices have fallen off since last May more than in any similar length of time since 1922, is another sign unfavorable to the continuance of the bull market. In the past such reactions in the bond market have always been followed by trade reaction or depression. When the flow of investment funds into the bond market stops, industry is no longer adequately financed and hence depression follows. There is no evidence that industry has ever secured through the stock market funds sufficient to carry it along successfully. It must be admitted, however, that today corporations are better financed, better managed, and have a higher standard of ethics than ever in the past. Furthermore, people are learning the science of investment. All of these forces tend to stabilize industry on a high level.

Despite these facts, however, one must remember that over-confidence has always been followed by fear. If interest rates begin to fall, it will not be a favorable but an unfavorable sign, as far as the stock market is concerned. The chances are that the volume of brokers' loans will contract before autumn of this year and that trade will feel the tightness of money. If the stock market begins to decline, business will slacken, for most business men use the stock market as a barometer, and when it shows an appearance of weakening, they curtail their activities.

While it appears, then, that depression is in the offing, it will probably be short-lived because of the fact that the underlying supports of industry are at present so well buttressed. The chances are that the decline in the stock market will not exceed 15 to 25 per cent of the present value. When this deflation has taken place, business will be left in a thoroughly secure position.

The second speaker of the evening, Mr. Thomas S. Holden, of the F. W. Dodge Corporation, dealt with "The Building Outlook." He began by pointing out that the construction contract record for February, 1929, showed a marked decline from the February, 1928, total. While declining building volume was rather expected in the early months of 1929, the February decrease was more severe than anything that had been seen for some years. This calls for careful appraisal of the general situation in order that too much significance may not be given to the record of a single month.

During the period 1921–1925, annual increases in construction expenditures averaged 888 million dollars; since 1925 the average annual increase has been but 147 millions, about one sixth the rate of the former period. The 1921–1925 period was one of rapid expansion in building, general business, and industry, supported by vastly augmented credit facilities and expressing itself in widespread prosperity and a distinct advance in living standards. Per capita construction expenditures rather more than doubled between 1921 and 1925. But the accelerating force that produced rapid expansion appears to have spent itself, at least for the time being. The secular-trend curve for construction volume in the past ten years is an S-curve, with a period of gradual increase dating from 1925. Hereafter, increasing population would demand slowly increasing con-

struction expenditures; some new accelerating force tending toward expansion might also bring about greater per capita expenditures.

Since 1925 there have been definite evidences of cyclical movements in building. There was a general recession in 1927; some whole regions and some large cities have had continuous declines in building volume since 1925. More frequent adjustments of building supply and demand seem to be moderating the severity of the cycle, and there is some reason to believe that the period of the short cycle is a little shorter than it used to be. The current decline in building seems to be a downward phase of the cycle. The decline to date has largely consisted of reduced volumes of residential building, always the first to decline in a downward cycle movement. There is evidence that indicates the current movement to be an adjustment of residential building supply to demand.

Evidences of stability are seen in continuously increasing assets of building and loan associations, increased savings, stabilized construction financing, and in the absence of radical price changes in recent years. These stabilizing factors give support to the belief that the current decline need not be prolonged or severe.

The current movement seems likely to run as follows: residential building is likely to decline for some months; non-residential building (now being contracted for in larger volume than ever before) is also likely to fall off a little later; these two factors probably resulting in somewhat less building volume than in 1928. Public works and utilities projects (mainly civil engineering work) are in great demand and are recognized as desirable in large volume at times when private contracts are declining. This class of work is more likely than any other to increase this year, and might offset to some extent the probable decline in building.

The third speaker of the evening was Dr. Julius H. Parmelee, director of the Bureau of Railway Economics, Washington, D. C. The topic which he chose for discussion was "The Railroad Outlook." He pointed out that in considering the railway business, one must always keep in mind that it reflects directly all other types of business activity, for no business can get along without a certain amount of transportation.

For their revenues, the railroads are dependent primarily upon freight, and only incidentally upon passenger traffic, the passenger revenues constituting but about 15 per cent of the total of all revenues.

Charts which Dr. Parmelee distributed showed separately, for passenger and freight traffic, trend lines constructed on the basis of the records for years preceding 1915. After 1915 the number of passenger miles followed the trend line fairly closely until 1920. Since then this curve has dropped far below the trend, in 1928 being one third lower than in 1920, and even below the level of 1909. This great drop in passenger traffic has, of course, been due to the competition of the automobile.

The curve representing the ton miles of freight traffic has not only kept up to the trend indicated to 1915, but has actually risen above it. The years 1926–7–8 were record-breaking years for freight traffic. It is, of course, true that motor trucks and transportation by water have had some adverse influence on the volume of railway freight, but apparently as yet this influence has been almost

negligible, the total of competitive freight carried probably not exceeding 5 or 6 per cent of the freight carried by railways.

What are the indications for the rest of 1929? The volume of freight normally grows from 2½ to 5 per cent per annum. The freight traffic for January, 1929, however, shows a 7 per cent increase over that of January, 1928. Carloading figures for the first nine weeks this year are 4 per cent larger than for the corresponding period last year. A three months' moving average, however, indicates that at present there is a downward cyclical movement in the volume of traffic. All things considered, the chances are that the total volume of freight traffic in 1929 will be between 1 and 2 per cent larger than in 1928. This will probably cause an increase of freight revenues of a hundred million dollars. Passenger revenues will probably diminish by forty millions. Mail revenue will increase because of the additional allowance made by a recent decision of the Interstate Commerce Commission. The chances are, then, that total railway revenues for 1929 will be at least 100 million dollars larger than in 1928, and that net income will rise by 75 millions.

The railroads have been economizing greatly in recent years. Despite a decline in revenues and a rise in taxes, they cut their total expenses last year 143 millions of dollars. It is hoped that there will be some further reduction in expenses in the present year. The probabilities are that the rate of return on the investment of railways will be from 4.8 to 5 per cent in 1929, as compared to 4.7 in 1928.

It is likely that the railroads will buy somewhat more supplies and more equipment this year than they did last. The total outlay for materials and supplies, equipment, and other capital improvements will probably be about 2¼ billions in 1929.

The last of the four regular speakers on the program was Dr. Ralph C. Epstein, of the University of Buffalo. The title of his paper was "The Automobile Outlook." He reminded the audience that in 1905 the motor age was said to have arrived because twenty-seven automobiles passed the Union League Club in one afternoon. From 1903 to 1916 the rate of growth in the production of automobiles was remarkable, the increase being about 45 per cent per annum, and remaining practically constant during the entire period. Between 1916 and 1923 the proportionate rate of increase diminished remarkably. Since 1923 the total volume of automotive output has been almost stationary, except for cyclical movements.

What are the prospects for 1929? Magazine writers have expressed very optimistic sentiments concerning the outlook for this year. The production records for January and February have been far above those for the corresponding months of previous years. In analyzing these figures, however, it is well to keep in mind the fact that the production of Ford and Chevrolet cars accounts for more than half of the total output at the present time. The Ford factory has been accelerating its production ever since it opened last year. The Chevrolet Company has substituted a six-cylinder car for a four. Other passenger car producers have announced new models at a date earlier than normal. It may, then, well be true that the high record for January and February is indicative

mainly of these transitory forces, and that, therefore, the seasonal curve for 1929 is different from that characterizing the past few years. Of course, if this is not true, the outlook for 1929 is extremely favorable. Should the ratio of output for the first two months of this year to the year's total be the same as in previous years, then more than 5 million vehicles would be turned out in 1929.

The chances are, however, that the 5 million figure will not be reached. If it is, it will probably be because of the growth of the export business which has been developing at a very promising rate during the past few years. Exports, however, advance very irregularly, increases in recent years having run between 1 per cent and 59 per cent per annum. It is possible, however, that the export business may increase in 1929 by as much as 200,000 passenger cars.

In making estimates of the total volume of production, it is necessary to distinguish between the demand for new cars to replace cars not yet worn out and the demand to replace cars that are actually scrapped. The fact that many families are now coming to feel that they need two cars is also a material factor.

There is no question but that the total volume of the automobile production today depends very largely upon the volume of general business activity. It is likely, however, that during the next three years the production of automobiles will not average more than 4¾ millions annually. It is probable, also, that the percentage of net profits on investment will be materially lower in 1929 than in 1928, for competition seems to be becoming increasingly keen.

The discussion of the four papers of the evening was begun by Mr. W. C. Clark, economist and vice president of S. W. Straus and Company. Mr. Clark said that the thing that interested him most as regards its influence on the building outlook was the question of whether the failure of courage looked for by Mr. Clay would come in April or in September. If it did not happen until September, it would have very little effect upon the building record of 1929, as most building plans for the year will have been completed long before that date, and likewise it probably would have little effect on the record of 1930. Mr. Clark agreed with Mr. Holden in feeling that the secular trend of building will be moderately rising for some time to come. He was less confident, however, that we would escape a downward cyclical movement, due to the hardening of interest rates. Any downward movement, however, would be offset to a considerable extent by the following new developments in the building industry, as well as by the stabilizing influences discussed by Mr. Holden:

1. Large financing corporations have recently been organized, having as their purpose the furnishing of an abundance of ready capital for short-time building loans.

2. Large scale consolidations of banking, building, and real estate concerns have taken place, and they are putting the building business on a much firmer and more stable basis than formerly.

3. Much more of the building activity of the country is being financed by stock issues than was formerly the case, and this fact will prevent the high interest rates now prevailing from having so much influence on building as would otherwise be the necessary effect.

This stock financing does not noticeably affect the cost of senior money, but it

has a great influence upon the availability and the cost of junior remaining funds. The fact should be kept in mind, however, that because of the demand and supply situation, the three causes just mentioned are influential primarily in New York City alone.

The discussion was continued by Mr. F. Leslie Hayford, economic statistician of General Motors Corporation. He began by stating that he agreed with Dr. Epstein that the volume of automobile production is largely dependent upon general business, but he also expressed the opinion that this had not been so true a few years ago as it now is. In general, his present feeling was that Dr. Epstein was somewhat too pessimistic about the production outlook for 1929.

Mr. Hayford stated that he did not expect that production would continue throughout 1929 at the rate established for January and February, and that the output for the first quarter would probably constitute a larger proportion of the year's volume than usual. He believed, however, that not enough cars had been purchased in 1928 to satisfy the existing demand, and that this fact would tend to make 1929 production high. The chances are that the domestic demand for passenger cars will be at least as strong in 1929 as in 1928 and that the output of passenger cars and trucks for export will be greater than last year. Also, it is probable that dealers will increase their stocks carried over at the end of the year, since the stocks of a number of makes of cars were abnormally low at the end of 1928. There is every reason to believe that production for the first half of 1929 will break all records, and, while the second half probably will not be so active, the total for the year seems likely to be relatively high.

Mr. James S. Hughes called attention to the fact that the breaking point of courage, as regards stock purchasing, never occurs except in the months when oysters are in season.

Mr. K. W. Jappe felt that the fact that bank credit, which normally tends to expand at the rate of about 7 per cent per annum, had not recently been growing so rapidly is a sign that some depression in industry is likely to make itself felt soon and that this will occasion a reaction in the stock market.

WILLFORD I. KING

FELLOWSHIPS OF THE SOCIAL SCIENCE RESEARCH COUNCIL

At its recent meeting the Fellowship Committee of the Social Science Research Council awarded 29 research fellowships. The awards were distributed among the social sciences as follows: Anthropology, three; Economics, six; History, six; Political Science, four; Psychology, three; Sociology, three; Geography, one; Law, two; Miscellaneous, one.

The six Fellows selected from the applicants in Economics are:

Hamilton, Earl Jefferson (Ph. D., Harvard)

Assistant Professor of Economics, Duke University.

Project: Money, prices and wages in Castile, 1500–1660, especially as affected by the influx of treasure from the Hispanic Colonies of the New World; and American gold and silver production 1557–1660.

Study in Spain, especially in Castile.

Hohman, Elmo P. (Ph. D., Harvard)
Social Science Research Council Fellow 1928–29.
Project: A comparative study of American and European seagoing labor in the twentieth century, with special reference to the operation of the LaFollette Seamen's Act.
Study at Geneva and Far East maritime centers.

Hoover, Calvin Bryce (Ph. D., Wisconsin)
Professor of Economics, Duke University.
Project: The banking system of the Union of Socialist Soviet Republics with special reference to the problems of a "managed currency."
Study in Russia and the United States.

Johnson, Edgar A. J. (Ph. D., February, 1929, Harvard)
Instructor and Tutor, Harvard University.
Project: A study of British mercantilism of the eighteenth century.
Study in London.

Rauber, Earle L. (Ph. D., August, 1929, Chicago)
Student and Research Assistant, University of Chicago.
Project: Labor legislation as manifested in practice as compared with paper programs.
Study in Prague, Vienna, and Moscow.

Staley, Eugene (Ph. D., December 1928, Chicago)
Research Assistant, Causes of War Committee, University of Chicago.
Project: International private investments as a factor in international relations.
Study in London, Paris, Berlin, and perhaps Geneva.

The three Fellows selected in Sociology are:

Brown, Esther Lucile (Ph. D., June, 1929, New Hampshire)
Assistant Professor of Sociology, University of New Hampshire.
Project: Lycanthropy—the Loup Garou.
Study in Canada and France.

Lumpkin, Katharine Du Pre (Ph. D., Wisconsin)
Instructor in Sociology and Economics, Mount Holyoke College.
Project: A study of the social situation as related to conduct-problem children in dependent families.
Study in New York City.

Wirth, Louis (Ph. D., Chicago)
Assistant Professor in Sociology, Tulane University.
Project: Segregated areas and local communities in German cities: a comparative study in methods of social research as affected by recent developments in human ecology and social psychology.
Study in Germany.

These fellowships are designed to promote the development of research workers rather than to aid in the execution of specific research projects. It is hoped they may save to the field of productive scholarship promising doctors of philosophy

whose zeal for research and writing has been gradually ebbing under heavy and diversified teaching schedules and routine administrative responsibilities. Or again, by affording a year of travel and investigation, the fellowships may broaden and enrich the scholarship of the young men and women who have just completed their Ph. D. work. Such a year may enable a Fellow to push further his researches in the field which his Ph. D. thesis has opened up. It may simply result in his digging deeper within the narrow confines of his own discipline. This in itself is worth while. It may, however—and the Committee hopes that this will frequently prove to be the case—enable him to broaden his approach to the problem of his special interest. He may elect to use the year to study such phases of other social disciplines as will enable him better to understand the ramifications of his own problem. In this way he will become not only a more effective economist or historian, for example, but also a more effective social scientist.

Awards are made once a year. Applicants should apply to John V. Van Sickle, Fellowship Secretary, 230 Park Avenue, New York, New York, in time to get their formal applications in by December 1 for the awards beginning July 1 following. The Committee meets in January or early February and the awards are made known around the middle of February.

The fellowships are open to men and women, citizens of the United States or Canada, who hold the Ph. D. degree or its equivalent and who are not over thirty-five years of age. These requirements are not absolutely iron-bound, but exceptions are more and more infrequent. The general term of the fellowships is one year, though in exceptional cases they may range from three months to two years. The stipend varies with the requirements of the Fellow, such as his present salary, number of dependents, and amount of travel involved.

PROGRESS OF WORK IN THE CENSUS BUREAU

THE CENSUS BILL

The bill providing for the next, or Fifteenth Census, failed of passage at the last session of Congress, which terminated on March 4. Its fate was sealed as soon as it became evident that the bill for a reapportionment of representatives could not be passed over the opposition which developed in the Senate, as the sponsors of reapportionment would not consent to have the census bill taken up so long as no apportionment bill had been enacted.

A census bill substantially the same as that which failed of passage has been introduced at the present special session of Congress, but its fate again depends upon the willingness of Congress to make provision for the reapportionment. At this writing, the prospect for the enactment of both measures seems very favorable. If a census bill is enacted it is probable that the date of the census will be November 1, 1929, as that is the date favored by the agricultural interests and appears to be the best for a census of population, coming at the time of the year when, generally speaking, the people of the United States are most likely to be found at their homes or usual places of abode. If, perchance, the census bill

should not pass, the Bureau, as explained in the March issue of the JOURNAL, will have to fall back upon the act of 1919, which requires the census to be taken in January and is in other respects an unsatisfactory law as applied to present conditions.

It seems fairly certain that the bill will provide for a census of unemployment, and the Bureau is already giving consideration to the question of the best method of obtaining satisfactory and reliable statistics on that subject. It is not certain that it will suffice to add one or two general questions to the population schedule. It may be advisable to cover the subject on a separate supplemental schedule carrying several questions to be filled out by the enumerator of the population whenever he finds any person having a gainful occupation but not employed at the time of the census. How many and what questions it will be necessary to ask in order to obtain the desired information regarding the extent or degree of unemployment, is, however, a matter yet to be determined.

One difficult problem that always confronts the census taker, and one which with the increasing mobility of the population is becoming ever more difficult, is that of enumerating the people in the localities in which they belong, or of including in the population of any given state, city, or other political unit, those classes which constitute its population for the purposes of the census. In the instructions to enumerators at recent censuses it is stated that the law requires that the people shall be enumerated at their usual place of abode.

The question, however, of who shall be included in the population of any given locality is, in fact, one to which the law gives no very definite answer. The decennial enumeration of population is required by the Constitution as a basis for the apportionment of representatives. But the Constitution says simply that representatives shall be apportioned among the several states according to "their respective numbers," counting the whole number of "persons in each state," excluding Indians not taxed. The population of a state or a city, then, according to the Constitution consists of the persons who are in the state or in the city. This does not help us much. What persons are "in the state" in the sense which these words are used in the Constitution? The earlier census acts use the term "number of inhabitants," the marshals being instructed "to cause the number of inhabitants within their respective districts to be taken"; and the oath which was prescribed for the assistant marshal required him to make an enumeration of "all persons resident" within the division. The term "census of population" rather curiously does not appear in any census act prior to that providing for the census of 1880. The law, however, has never supplied a definition of "population" or of "inhabitant" or of "resident" or of "usual place of abode."

The instructions to enumerators in 1910, and again in 1920, defined the usual place of abode as being the place where a person "may be said to live or belong or the place which is his home." Perhaps it might be best defined as the place which a person would name in answer to the question "Where do you live?" It is not a question of legal or voting residence.

Assuming provisionally that we have a fairly definite idea of what constitutes the usual abode and of what classes of people should be included in the population of any locality, the difficult practical question arises of how we shall manage to

enumerate the people at their usual places of abode when they are not there. The instructions to enumerators in the past have contained rather detailed directions as to what to do in those cases where an individual is absent from home on the date of the census, the general principle being that information regarding the absent member of the family should be supplied by other members of the family, so as to include him in the population of the locality where the family resides; and that he should not be included in the population of the locality where he might happen to be on the date of the census, *unless* it appears that there is no person at his home or usual place of abode who will give his name to the enumerator, in which case he must be enumerated in the place where he is. This is where the principle of enumeration at the usual place of abode does not work, and the *de facto* population supplants the *de jure*.

While the instructions to the enumerators in prior censuses gave careful consideration to the case of the individual who is away from home, they paid little or no attention to the case in which the entire family is away from home. In former days absent families were perhaps not so very numerous, but at the present time they are coming to be altogether too common for the comfort of the Census Bureau. Very many farm families move into town or the city for the winter, and, of course, large numbers of families go to Florida or California or other resorts for a portion of the year from all parts of the United States. How shall we manage to enumerate these families at the place where they belong and not enumerate them at the place where they do not belong, so as to include them in the population of the former place and exclude them from the population of the latter?

In 1920 some use was made of a so-called absent-family schedule. The idea was that the enumerator coming to a house that was closed and not being able to get the required information from the neighbors or other local sources was to arrange through the supervisor to have a schedule mailed to the address of the absent family with instructions to fill it out and return it to the supervisor.

Another possible plan of procedure would be to enumerate all families in the places where they are, but to use a non-resident family schedule for families whose usual place of abode is somewhere else, which schedule would be filled out in the presence of the family, returned to the Census Bureau with the other schedules and there be allocated to the place in which the family belongs. This method of procedure would probably ensure a more complete enumeration with less danger of omissions and of duplications and a more accurate filling out of the schedules; but it would entail a vast amount of work for the Bureau in allocating the non-resident family schedules to the districts in which they belong and would delay the final announcements of population.

If the census is taken in November this problem of the absent family may not be very serious, as that is a season when the population is very generally at home. But if it is taken in January, as the old law requires, there is reason to apprehend that there will be a large number of families absent from their usual places of abode when the census taker calls.

J. A. H.

MISCELLANEOUS NOTES

Meetings of the San Francisco Chapter.—The tenth dinner meeting of the San Francisco Chapter was held at the Bellevue Hotel, San Francisco, on Friday evening, January 25, 1929. Forty members and guests were present.

The assembly was called to order by our President, Holbrook Working, of the Food Research Institute, Stanford University. Dr. Working introduced our National President, Dr. Edwin B. Wilson, of the Harvard School of Public Health. Dr. Wilson consented to act as chairman for the evening, and made many happy comments upon the speakers and talks which followed.

Prior to the discussion of the general topic of averages, the proposed revision of the calendar was explained by Mr. C. L. Queen, of Lybrand, Ross Brothers, and Montgomery, who spoke on "How Do You Tell What Day This Is?"

The general topic of the meeting was "Fitting the Average to the Job." The first speaker was Professor John B. Canning, of Stanford University, who spoke on "Statistical Theory as a Basis for Choice of an Average." Professor Canning's remarks are outlined in the following paragraphs.

The scope within which selection is possible is limited only by our general notion of an average. The broader notions include any systematically determined representative value or any value that is convenient in the matter under investigation. The narrower notions restrict selection to a value "thoroughly representative" of the whole series or to a value expressing a "central tendency." The broader notion is the more convenient and the more accurately descriptive of practice, because a value lying at or near the maximum or minimum observed values may possess a greater critical importance in a given investigation than any value lying near the center of the range, and no reason can be assigned (or has been assigned) for excluding values outside the range.

The selection of an average includes the decision as to whether any average is significant at all for the matter under investigation. Any average may be a too crude simplification, for example, "expectation of life," "probable lifetime," or "most probable lifetime" in computing life annuities, and in other fields, average temperature, average rainfall, or average humidity.

Assuming a range of possible choice and the appropriateness of some one or more averages, no basis of choice other than convenience suggests itself. "Convenience" includes two principal notions, (a) possibility of achieving a desired effectiveness, and (b) labor saving in process and in interpretation. Some but not all convenience arises out of the mathematical properties of the formulas of the several averages. For instance, in weighing an object with beam balances having arms of unequal length, one reverses the position of object and poises and takes the geometric mean of the two poise readings, but in weighing without respect to error in the scales one employs the arithmetic mean of many readings to find the most probable value for a single reading. When aggregate quantity in a standard container is significant, tolerance limits should be measured from the arithmetic mean; but if relative deviation is important, as in the case of the potency of a drug in a container, potency tolerance limits should be expressed as logarithmic differences from the logarithm of the standard potency measure, *i.e.*, the standard potency should be the geometric mean of the limits. The geometric mean has a bias if calculated from values of a continuous variable determined by measurements on an arithmetic scale. The location of a tool room in a shop and the tool room layout should have the property of the median, so that the aggregate travel distances to the tools should be a minimum. Some averages

are convenient as measures of dispersion (averages measured from an origin other than zero).

Interrelations of certain averages are not yet satisfactorily investigated. Thus, it is commonly said of two positive values that their geometric mean equals the geometric mean of their arithmetic and harmonic means, but that this relation is not general to series of more than two terms. On the contrary it can be shown (a) that the two terms constitute but a special case of a much more general series of n terms, for if the logarithms of a series of n values can be formed into a frequency distribution that is symmetrical about the logarithm of the geometric mean of the series it can be shown that the relationship of the three means described above holds for this series; (b) that indefinitely many series of n terms other than those described above will satisfy the condition, for $n-1$ values can be chosen at random and solution be made for the several values of the nth term that will satisfy the relation; and (c) that if any series of n terms satisfies the relation, indefinitely many other series may be derived from it that also satisfy the relation.

Selection of an average is not limited to a choice among previously defined averages. Given the desired set of properties it is possible to invent formulas that possess the properties. Witness the recent work of Fisher and others in index numbers.

The last speaker was Robert W. Bachelor, of the Division of Analysis and Research of the Federal Reserve Bank of San Francisco. Mr. Bachelor spoke on "Convenience and Simplicity as the Basis for the Choice of an Average." A summary of his remarks follows:

Many volumes have been written about averages, but convenience and simplicity as a basis for choice have apparently not been considered. Discussions have centered around the theoretical differences between averages without giving a thought to the relative costs of calculation. The actual differences in results in a particular problem may be slight, but the difference in the expense of calculation may be great.

The choice of an average depends almost entirely upon the problem at hand. There are many kinds of averages and each has its place, but some are laborious to calculate and some are not easily understood by the consumer of statistics. An average may be regarded as a measuring instrument by means of which a series of items is judged and its characteristics ascertained. On the other hand, it may be a mere theoretical abstraction with very little meaning.

An average represents something which is typical of the data, and in many cases we have to use several averages to describe the various characteristics of the series. We choose the form of average which will best bring out the relationships which we want to show. In certain economic data, if we wish to study very long period fluctuations or cycles we may fit a straight line as an average of the series. If we are studying shorter period swings we will fit a curve which follows the data more closely, and if we wish to study seasonals we use an annual average or a twelve months moving average. In many cases the average may be drawn in free hand, for preliminary studies.

The standard deviation is important as a measure of the behavior of the series and should be used to supplement the other averages which describe the data. Weighted means, geometric, harmonic, and other averages which are expensive to calculate and little understood by the average user of the results should be avoided except where absolutely necessary.

Index numbers, whether of production, commodity prices, security prices, or other things, are merely averages which save the labor of the reader in examining all the original quotations. Certain index numbers of security and commodity prices must be available quickly. They are hurriedly put together without respect to the most proper methods of averaging, and reach the user to serve in his operations, while the

more scientifically constructed numbers, mathematically precise, are available only for use in historical analysis, due to the extended labor of calculation.

Moving averages are important in smoothing a series to show the major movements and trends, free from the frequent accidental and erratic fluctuations.

Averages are very important in the theory of probability and in the normal curve of error. In many cases the normal curve will serve to describe the series and may be far better than the original data for the solution of practical problems. If we determine that the probabilities are that the normal curve gives a reasonably good fit, we may employ it to describe our distribution.

On March 22 the San Francisco Chapter held its eleventh dinner meeting at the Bellevue Hotel, attended by an unusually large number. The guest speaker of the evening was Dr. Edwin B. Wilson, president of the American Statistical Association. His subject was "The Economic Value of Vital Statistics." The meeting was very successful. It is hoped that the manuscript of Dr. Wilson's address will be available for publication at a later date.

Activities of the Chicago Chapter.—The Chicago Chapter held a meeting on February 19 which was attended by nearly sixty people. Professor Edie of the University of Chicago was the speaker.

Professor Edie believes that London essentially is still the financial capital of the world. Directors of the Bank of England have a keen knowledge of international affairs and do not allow local conditions to obscure their understanding of world developments. The contrast between the depth of vision of American and English bankers is illustrated by their respective explanations of the 12 per cent decline in prices during the period 1925 to 1927.

American bankers were characteristically provincial, limiting their explanation to conditions within the United States. They regarded the decline in prices as merely the result of the country's increased industrial efficiency—an explanation which was shortly proved inaccurate by the subsequent rise in prices. American bankers apparently were not aware of the economic law which causes the price levels of all countries under the gold standard to move together.

English bankers viewed the price decline from an international angle and came to the conclusion that it was the result of a temporary credit stringency caused by the return of European nations to the gold standard. They recognized that the increased gold reserves required by the return to the gold standard caused money rates throughout the world to increase and consequently caused prices to decline.

The international-mindedness of the directors of the Bank of England is further illustrated by the responsibility which they feel in stabilizing world prices. The officials of the Bank of England have turned a deaf ear upon the repeated pleas of English industries that an easy money policy be established at home to aid in bringing business out of its depressed state because they believe that such a policy would not be in harmony with their more important function of stabilizing prices.

There is a general feeling in England that the United States does not take its responsibility as custodian of the largest gold reserves in the world seriously enough, and that it will use its financial power to further its own ends at the expense of other nations. In particular, English bankers fear that the United States, instead of devising a means of curbing speculation which will not disturb the rest of the world, will attack the stock market by raising its rediscount rate. Such a step, of course, would cause a decline in the price level of the world and, in the opinion of the English bankers, would prove that the United States was not qualified to control the huge amount of gold which it has acquired.

Professor Edie believes that there is apt to be a gold shortage during the next decade because many European countries, desiring more gold as a preparation for war, as a means of increasing public confidence in their currencies, and as a method of building up their prestige as financial centers, have begun to hoard the yellow metal. A development which is worrying English bankers particularly is the desire of France to put gold coin back into circulation. In order to do this France will have to withdraw her huge balances in London and in New York. If these withdrawals are made, the gold reserves in both London and New York will be seriously impaired, money rates will rise, and the world price level consequently will decline.

In an effort to thwart this plan, England has persuaded the League of Nations to make a study of world gold supplies. English bankers expect that this study will reveal the need of conserving gold and hope that public opinion will then force France to abandon her plan. Unless this report is completed in the near future, however, it may be too late to secure the desired results, because France already has begun to mint gold coins.

A decline in the price level of as much as 10 per cent, of course, would seriously disturb conditions throughout the world, because in effect it would add 10 per cent to all loans contracted before the reduction in prices occurred. Consequently, English bankers are of the opinion that if the price level actually declines, England will have to repudiate a substantial part of her war debt to the United States.

On March 28 another meeting of the chapter was held with an attendance of about thirty persons, who were greatly interested in a talk by Professor William F. Ogburn on "Some Statistics of the Modern Family." The basis of Professor Ogburn's paper was the analysis of Census results and the conclusions to be drawn from them. The particular topics dealt with were the relation of income to marriage, the influences of city life upon marriage, a comparison of rural and urban rates of divorce, and other factors affecting the modern family. The meeting was presided over by Professor Ernest W. Mowrer, of Northwestern University.

The Annual Meeting of the Boston Chapter.—The second annual meeting of the Boston Chapter was held at the Boston City Club on April 12 and was attended by forty-one members and guests. Dinner was served at 6.30 P. M. At the business meeting, the following were elected as officers for the ensuing year:

President: Charles F. Gettemy, assistant Federal Reserve Agent, Boston Federal Reserve Bank

Vice-President: Kenneth B. Emerson, statistician, United Shoe Machinery Company, Boston

Secretary-Treasurer: Roswell F. Phelps, director, Division of Statistics, Massachusetts Department of Labor and Industries, Boston

Counsellors: Joseph B. Hubbard, editor, *Weekly Letters*, Harvard Economic Society, Cambridge, Massachusetts; Hermann F. Arendtz, United Business Service, Boston

Following the business meeting there was a speaking session at which Mr. Charles F. Gettemy presided. The general subject for consideration by the four speakers was "The Regularization of Industry."

The Honorable Ralph O. Brewster, formerly governor of Maine, who had recently been in conference with President Hoover, discussed "The National Plan for Regularizing Industry," and stated that the establishment of national and state agencies for securing authentic data relative to industrial trends is the first major step in any program that offers a sound and permanent solution of the problem.

Dr. William T. Foster, director of the Pollock Foundation for Economic Research and co-author of *The Road to Plenty*, spoke on the subject "Stabilization of Employment," and in connection therewith discussed the recent action of the Federal Reserve Board in its attempt to regulate speculation. Dr. Foster expressed his opinion that any plan to stabilize employment could not be successful if it were not in accord with the policy of the Federal Reserve Board.

Mr. J. A. Garvey, personnel manager of the Dennison Manufacturing Company, spoke with reference to the several measures adopted by that company in its endeavor to stabilize employment in its own plant, through the reduction of seasonal orders, increase in the proportion of non-seasonal orders, advance planning for the production of holiday and other stock items, the building up of "out-of-season" items, and the distribution of long-time orders and out-of-season items throughout the year. As a result of such measures, the unemployment fund established by the company has been more than adequate to meet requirements, even during periods of general industrial depression.

Mr. Charles P. Howard, chairman of the Massachusetts Commission on Administration and Finance, in discussing "The Building Construction Program of the Commonwealth," stated that in planning for the construction of buildings, highways, and other public works for the Commonwealth, not of an emergency nature, it was found possible, in some instances, to defer the placing of contracts, not only so as to secure more favorable terms, but also to offset, in some measure, depression in the building trades.

Activities of the Detroit Chapter.—The Detroit Chapter has held two successful meetings since the first of the year. The first, held on January 16, with Professor Clare E. Griffin as chairman, had for its general subject "Business Conditions in 1919." Mr. John F. Scoville, statistician of Chrysler Motors, in making his predictions for this year, took occasion to refer to several of his painstaking studies of the automobile market, demonstrating his findings with lantern slides. The money and security markets were covered by Mr. Albert J. Hettinger, Jr., whose analysis of future conditions was most informative and stimulating. Finally, Colonel Leonard P. Ayres, well known to members of the Association, summed up business conditions generally in his usual masterful fashion. There was a record-breaking attendance of 116 as a compliment to Colonel Ayres.

The second meeting of the year was held on March 13, with a more normal attendance of 26. It turned out to be University of Michigan night, for both speakers were of the faculty of its School of Business Administration. Professor Carl N. Schmalz spoke interestingly of his recently published report, *Standards of Performance in Department Stores*, and Professor Clare E. Griffin reviewed comprehensively his monograph on *Sales Quotas*.

Activities of the Pittsburgh Chapter.—On the evening of January 22, the Pittsburgh Chapter of the American Statistical Association held a dinner meeting at the Faculty Club of the University of Pittsburgh. The speaker was Mr. Thomas S. Holden, of the F. W. Dodge Corporation. Despite unpleasant weather, the meeting was very successful, but the weather conditions limited the attendance to twenty. Mr. Holden spoke of conditions in the construction industry during the last five years and estimated prospects for 1929. He related construction indexes to barometers of business conditions in other fields and illustrated with the measures used by the F. W. Dodge Corporation. He illustrated the need of employing both statistical measurements and economic judgments by showing what errors in estimates might have been made in recent years by employing either exclusively.

On March 25, fifteen members of the Association attended a meeting at which Dr. O. C. Stine, in charge of the Division of Statistical and Historical Research, Bureau of Agricultural Economics, led a discussion regarding the significance of agricultural prices in the interpretation and forecasting of agricultural problems. He referred in part to the particular problems of western Pennsylvania, Ohio, and West Virginia.

Meeting of the Austin Chapter.—Twenty-four members and guests attended the meeting of the Austin Chapter on March 13. Dr. W. J. Reilly, marketing specialist in the University of Texas Bureau of Business Research, presented the results of his studies on "A Method for Interpreting the Reliability and Accuracy of Small Samples," a method worked out by Dr. Reilly in connection with the analysis of questionnaires secured in market studies conducted by the Proctor and Gamble Company under his supervision. By breaking up the samples into groups of ten and calculating certain statistical measures for sequences consisting of successive groups of five consecutive blocks, significant changes in the data from which the sample is drawn will be revealed by variations in the statistical measures calculated. The discussion of the method was led by Dr. L. A. Jeffress.

Meeting of the Los Angeles Chapter.—The Los Angeles Chapter met on the evening of March 27, 1929, and elected the following officers for the remainder of the current year:

President: Arnold T. Anderson, the Security-First National Bank
Vice-President: J. V. Carson, the Title Insurance and Trust Company
Secretary: Ira N. Frisbee, University of California at Los Angeles
Treasurer: Harold E. Craig, the Security-First National Bank
Editor: George J. Eberle, Eberle Economic Service
Counsellors: Fred Knowles, Los Angeles Police Department; George G. Ellis, Bank of America

A paper was presented by Mr. Harold E. Craig on "The Nature and Purpose of the Monthly Business Review."

United States Bureau of Labor Statistics.—During the past quarter the Bureau of Labor Statistics began field work on wage investigations of bituminous coal and oil wells and pipe lines. Preliminary results will be published in the *Monthly Labor Review*, and the full report will be later issued in bulletin form.

An investigation of the hazards of spray painting, with special reference to the existing practices of protection and prevention in establishments using spray painting methods, has been completed and the results published in the *Monthly Labor Review*. A similar study of the use of radium and radioactive substances in industrial processes is also almost ready for publication.

A handbook of Labor Statistics for 1929, summarizing all the publications of the Bureau for 1927 and 1928, has been compiled and is now in press. This handbook is similar in form to the one issued in 1927.

A wage history of the United States is now being compiled by the Bureau. This volume will present for certain representative industries and occupations the course of money wages from the earliest Colonial records to the present time. The data from 1840 are derived entirely from the Bureau's own investigations, the published results of which have in many cases been long out of print.

Activities of the Bureau of Agricultural Economics.—A new section to develop work on tobacco has been established in the Bureau. This new section has been made

possible by the passage of The Tobacco Stocks and Standards Act, on January 14, 1929, which authorizes and directs the Secretary of Agriculture to compile and publish quarterly reports of the stocks of leaf tobacco in the United States in the possession of manufacturers, warehousemen, brokers, etc. The Act also authorizes the Secretary to establish the classification to be used in reporting leaf tobacco stocks. An appropriation of $30,000 was made in the first deficiency bill, passed March 4, to administer the Tobacco Stocks and Standards Act until June 30, 1930. Later expansion of the activities of the Tobacco Section to include a rather broad program of statistical and economic research is contemplated. The new section will be in charge of Chas. E. Gage, who will be assisted by F. B. Wilkinson and J. V. Morrow.

The Agricultural Appropriation Bill for the next fiscal year also makes available $20,000 for inspection work on tobacco similar to that now conducted on fruits and vegetables, hay, butter, eggs, meat, etc. Plans for the development of the tobacco inspection service have not yet been made, but in all probability this work will be developed as far as practicable in coöperation with state agencies.

The Bureau and the New Jersey Agricultural Experiment Station are coöperating in a study of the consumption of dairy products in northern New Jersey. The study consists of two parts: (1) a complete census or enumeration of the consumption and production of dairy products in six selected townships containing small towns or villages, as well as rural areas, by individual families and producers; (2) a sampling study of the consumption by 2,000 individual families selected by type in metropolitan New Jersey. Data on factors related to the consumption of dairy products, such as composition of family, location, income, occupation, and race or nationality, are also obtained.

There has just come from the press *United States Department of Agriculture Statistical Bulletin No. 25, Dairy Statistics.* There has recently been issued a report, *United States Agricultural Trade with the Philippines.*

Revision of Indexes of Employment and Payrolls of the Federal Reserve Board.—The Federal Reserve Board's indexes of employment and payrolls are now in process of revision. The revised indexes include data for a larger number of industries than those now published by the Board and are being computed on a 1923–25 base with weights based on figures from the census of manufactures for 1923 and 1925, adjusted to allow for 1924. As in the Board's present index, index numbers of employment and payrolls in individual industries are adjusted to the relative level indicated by the census of manufactures.

Criminological Statistics.—The Editor is in receipt of a *Guide for Preparing Annual Police Reports*, published by the Committee on Uniform Crime Records, International Association of Chiefs of Police. It has been sent to police departments in the expectation that it will be of timely aid in preparing annual reports for 1928. The final report of the committee, dealing with forms and procedure for compiling crime records, will appear at a later date. Copies may be secured from Bruce Smith, Director, Committee on Uniform Crime Records, 261 Broadway, New York City.

The German Insurance Science Society.—The German Society for Insurance Science held its annual meeting on April 10, 1929, at its headquarters, Landshuter Strasse 26, Berlin, Germany.

Dr. Hanns Dorn, of the Technical University of Munich, is the president of the Society and Dr. Alfred Manes, honorary professor of Insurance and director of the Insurance Institute at the Technical University of Berlin, is the general manager.

The Society is organized in four sections: Insurance law and economics; Actuarial methods; Insurance medicine; and Insurance education. At the close of 1928, the Society had 452 corporate members and 1,228 individual members. A number of American insurance companies are corporate members. Mr. N. Albert Linton, of the Provident Mutual Life Insurance Company, Philadelphia, is a member of the Board of Directors of the Society, and Mr. E. W. Kopf, of the Metropolitan Life, is a member of the Executive Committee of the Insurance Law and Economics Section. The Society's foreign membership embraces 38 countries.

In addition to its quarterly *Journal*, the Society also publishes individual monographs and a supplement on actuarial and statistical mathematics.

National Safety Council Form for Study of Fatal Accidents.—The National Safety Council has prepared a form for use in state bureaus of Vital Statistics for the description of the more than ninety thousand deaths which occur from accidents each year in the United States. At the present time, statistics for fatal accidents in the United States are presented each year only under the titles of the International List of Causes of Death, and these give information chiefly on the means of accident. The new form drafted by the Safety Council will enable state registrars to secure the following additional facts: the results of the accident, number of dependents left by the deceased and the class or kind of accident, whether it occurred in the home, in industry, as the result of a vehicular accident, or from other public hazards. For home accidents, the means of injury are specified, whether the accident was due to a fall, to burns and scalds, asphyxiation, poisoning, or cuts or scratches. For industrial accidents, practically the same information will be gathered for fatalities reported to the state bureaus of vital statistics as are now collected by workmen's compensation commissions. It is proposed to show the number of industrial accidents involving machinery, vehicles, explosions, electricity, fires and hot substances, poisonous and corrosive substances, falling objects, the handling of objects and the use of hand tools, etc. For vehicular accidents it is purposed to collect facts as to whether the person killed was the pedestrian or driver, whether the accident was the result of a collision, who drove the vehicle, the age and sex of the driver, and other important information needed for accident prevention purposes. For public accidents not involving a vehicle, the statistics will attempt to show whether death resulted from a street or a sidewalk accident, in buildings or structures, by drowning, through the use of firearms, and through other public accidents not involving a vehicle. On the reverse side of the form a schedule of definitions is given which will guide local and state registrars in the classification of accidents.

When this system goes into full effect in each of the states, it is hoped that the resulting statistics will provide the basis for a broad-gauge accident prevention movement directed at accidents in the home, in industry, and on our streets. The form was drafted by a joint committee representing the Statistics Section of the National Safety Council and the Section on Vital Statistics of the American Public Health Association.

The form should be of interest to insurance men, and copies may be obtained from the Statistical Division of the National Safety Council, 108 East Ohio Street, Chicago, Illinois.

PERSONAL NOTES

Allyn Abbott Young

No heavier loss could have fallen upon American statisticians and economists than the untimely death of Allyn Young. He was a leader in both groups and both expected great things from him in the decade to come. For long years he had given all too generously of his time to fellow workers. At last he was seeking to concentrate upon the intricate problems which intrigued his own mind most, and with which he seemed ideally equipped to deal. At the height of his powers, and at the outset of what should have been his harvest season, he died suddenly.

Young was born at Kenton, Ohio, in 1876. His father, Sutton E. Young, had the gift of public speaking, and took a leading part in the free-silver agitation of the middle 1890's. The boy must have been precocious, for he graduated from Hiram College at eighteen. He was also versatile: he became a printer and an organist, earning money with which to do graduate work at Wisconsin. In the fall of 1899 he was sent by Professor Ely to the Section of Analysis and Research over which Walter F. Willcox presided during the Twelfth Census—a group including Joseph A. Hill from Harvard, Thomas S. Adams from Johns Hopkins, David L. Wing from the Massachusetts Institute of Technology, Robert R. Kuczynski from Munich, and perhaps half a dozen others, myself among them. Young charmed all of us, as he charmed men throughout his life. And he won his spurs by an admirable study of "The Comparative Accuracy of Different Forms of Quinquennial Age Groups," printed in Volume VII of this JOURNAL.

Returning to Wisconsin after about a year in Washington, Young took his doctor's degree in 1902 and began his work as a teacher of economics. Presently he passed on to Western Reserve University and then to Dartmouth College. In 1906–11 he was head of his department at Stanford, with Thorstein Veblen, H. A. Millis, and A. C. Whittaker as leading colleagues. He next taught at Washington University, St. Louis, for two years, and in 1913 began a seven-year term at Cornell.

If all this moving about interfered somewhat with Young's researches, it brought an uncommonly wide range of contacts with his fellow workers, fitting him admirably for the self-sacrificing service he rendered as secretary-treasurer of the American Economic Association in 1914–20. In 1917 he was elected president of the Statistical Association. Appropriately to the times, his presidential address dealt with "National Statistics in War and Peace" (Volume XVI of this JOURNAL). The war theme was much in his mind, then and later. For Young had been one of the first to join in the work of economic mobilization and he remained one of the most persevering in striving for a better international understanding after the armistice. He served the War Trade Board as Director of Research in 1917–18, and the American Commission to Negotiate Peace as chief of the Division of Economics and Statistics in 1918–19. While president of the Economic Association in 1925, he delivered an address on "Economics and War"; he helped his friend Jeremiah Smith as economic counsellor on Hungarian problems; he was one of those interested in the Council on Foreign Relations, and to the end he gave keen attention to the processes of world readjustment.

Resuming university work after the Peace Conference in Paris, Young was called to Harvard in 1920. There he taught some seven years, making each year a deeper impression upon his colleagues and his students. When an American was thought of in 1927 to take the chair which Edwin Cannan had distinguished at the London School of Economics, all his fellow workers felt that the honor was rightly conferred upon Young. In England as at home, he won men's hearts and commanded their admiration.

For economic investigation Young was remarkably endowed. He united in rare measure mathematical powers, historical learning, and philosophical grasp. Indeed, his versatility was an ever-lurking temptation to disperse his attention. Many were the colleagues and the students who sought his critical and his constructive advice. Because his range was so wide and his insight so quick, as well as because he loved to help others, Young found it difficult to say "No." Few American economists of his generation are represented in so many books which bear other men's names. And Young's prefaces written to help others get a hearing would make a slender volume. His occasional papers, partly collected in the book which he published in 1927, *Economic Problems New and Old*, are further testimony to the catholicity of his interests and the stretch of his powers. They range from highly technical essays on index numbers to close critiques of economic concepts and the weighing of practical issues. Besides this volume we have the penetrating *Analysis of Banking Statistics*, reprinted from the *Review of Economic Statistics*. All of us value his published work highly, for though slender in bulk, it is of rare quality. But Young was not satisfied. He felt, and felt with increasing intensity as the years went by, that he must carry through the larger projects—particularly a systematic study of monetary problems—on which he had long been working. Those who saw fragments of the investigation could not but sympathize with his longing for time to finish what he sketched. He had begun to cut some of the bonds which tied him to miscellaneous tasks when the quick end came.

W. C. M.

William Horace Davis

Dr. William H. Davis, chief statistician for Vital Statistics, Bureau of the Census, died in Washington, D. C., January 8, 1929. He was born in Holyoke, Massachusetts, July 21, 1871, was a student at Amherst 1889–90, and was graduated from Harvard College in 1893 and from the Harvard Medical School in 1887. He studied in Vienna in 1899.

He was surgical house officer of the Massachusetts General Hospital in 1898, practiced medicine in Boston from 1900 to 1916, and from 1908 to 1916 had charge of vital statistics for the Boston Health Department. On September 16, 1916, he was appointed chief statistician for the Bureau of the Census.

Dr. Davis in virtue of his work and position had come to be recognized as one of the foremost vital statisticians for this country. In 1917 he represented the government at a conference in Ottawa, when the Canadian officials invited him to give expert advice on registration problems. In 1920 he served as a representative of the Bureau at the International Conference in Paris which revised the International List of Causes of Death. In 1925, on invitation from the League of Nations, he joined a tour of medical statisticians from fourteen countries and visited the registration offices of seven European countries. In 1926 he was appointed by the International Statistical Institute (of which he was a member) to serve on a commission to revise the International List of Causes of Death in 1929.

In 1927 Dr. Davis again went abroad to bring about greater uniformity in the compilation of vital statistics in the several countries. This was under the auspices of the Health Section of the League of Nations.

Dr. Davis was a tireless and an ardent worker in the field of vital statistics. He effected many improvements in the Vital Statistics reports of the Bureau of the Census and was especially zealous in his efforts to extend the death and birth registration so as to cover the entire area of the United States—an ambition which had been all but realized at the time of his death. He contributed many articles on vital statistics to

health bulletins and medical journals. His great activity in public health circles made the Division of Vital Statistics and its work of great interest to the country as a whole, and the Bureau of the Census has suffered a great loss in his death.

He was married to Miss Mabel A. C. Johnson of Milton, Vermont, on December 3, 1898. His widow and six children survive him.

J. A. H.

William Chamberlain Hunt

William C. Hunt, retired chief statistician for Population in the Bureau of the Census, died in Washington, D. C., on March 5, 1929. He had been in ill health for several years and had retired from the Census Bureau in June, 1925.

Mr. Hunt was born in Charlestown, Massachusetts, on December 20, 1856, the son of Samuel C. and Sarah K. Hunt. He was graduated from the Charlestown High School in 1875. He was connected with the Massachusetts state censuses of 1875 and 1885, and with the Massachusetts Bureau of Labor from 1883 to 1889. His connection with the United States Census dates from the year 1889. He was made chief statistician for Population in 1899. He had charge of the population censuses of 1890, 1900, 1910, and 1920; of the marriage and divorce inquiry of 1906; of the census of religious bodies taken in 1906 and again in 1916; of the special Oklahoma State Census of 1907; and of the New York State Census of 1905.

He wrote many of the census reports on population and other subjects. In collaboration with Carroll D. Wright he prepared the volume on *History and Growth of the United States Census*, published by the Department of Labor in 1900.

In 1921 he married Miss Caroline Arledge of Washington, D. C. His widow and two children by a former marriage, a son, William H. Hunt, and a daughter, Mrs. Arthur G. Hill, survive him.

J. A. H.

John B. Mitchell

John B. Mitchell, chief of the division of Vital Statistics in the Bureau of the Census, died on January 23, 1929, after a long and lingering illness. Mr. Mitchell entered the service of the Bureau as a clerk in 1900 and was advanced through various stages until he was appointed chief of the division in 1918, which position he held until his death.

J. A. H.

Martin J. Shugrue

Professor Martin J. Shugrue, thirty-nine years old, of the Department of Economics at the Massachusetts Institute of Technology, died in Brookline, Massachusetts, on April 4. He was a graduate of the University of Michigan. He came to the Massachusetts Institute of Technology in 1914. He was a member of the American Economic Association and the American Statistical Association and was joint author with Professor Davis R. Dewey of a work entitled *Banking in the United States*.

Mr. Carl Snyder was the guest of the faculty of Economics and Commerce and Administration of the University of Chicago at dinner on April 12. He led a discussion of the present financial and business situation in the United States, with particular reference to economic policies.

Dr. Horace Secrist has been granted an additional year's leave from Northwestern University to complete the research work he is doing for the special commission on Economics of Claremont Colleges in conjunction with Professor Frank A. Fetter and Dr. Keith Powlison.

Dr. T. F. Murphy, on February 5, 1929, was promoted to the position of chief statistician for Vital Statistics, filling the vacancy created by the death of Dr. William H. Davis. He was born on December 5, 1875, in Lewiston, Maine. He attended Bowdoin College and the Maine Medical School, and received the degree of M.D. from George Washington University in 1906. He was early connected with the Manufactures Division of the Bureau of the Census and with the Taft Tariff Board. Subsequently he became chief of division of Vital Statistics of the Census Bureau, assistant chief clerk, and has been in charge of the collection and compilation of current business statistics. Later, he was given supervision of the decennial Census of Religious Bodies, and in 1928 was again made chief of division of Vital Statistics.

Dr. John H. Cover has accepted an appointment at the University of Chicago, leaving his former post at the University of Pittsburgh.

Mr. F. E. Richter, formerly of the American Telephone and Telegraph Company, has accepted a position with W. H. Goadby and Company.

Dr. Hugh Carter, of the University of Pennsylvania, has been on leave this year making a study of public poor relief in Pennsylvania since 1875. His results are to be published this spring. The study is under the general direction of Mr. Ralph G. Hurlin, of the Russell Sage Foundation, and is financed by the Commonwealth Fund and the Public Charities Association of Pennsylvania. Dr. Carter will be on leave again next year. He will be occupied with research with the Committee on the Cost of Medical Care. Mr. Harry H. Moore is the director of this Committee.

Mr. Richard B. Mayer has been made assistant to the General Manager of the American Radiator Company.

Mr. Gerald B. Thorne, formerly of the University of Missouri, has been appointed associate agricultural economist in the Bureau of Agricultural Economics to assist in the price anlaysis and outlook work of the Bureau.

MEMBERS ADDED SINCE MARCH 1929

Albert, Charles F., de Saint-Phalle and Company, 11 Wall Street, New York City
Alexander, Harry W., 420 Lexington Avenue, New York City
Allen, Albert P., Illinois Bell Telephone Company, 212 West Washington Street, Chicago, Ill.
Allen, William S., H. K. McCann Company, 285 Madison Avenue, New York City
Banks, H. S., 241 East Mosholu Parkway, New York City
Barton, Lydia, The Vaness Company, 3700 Terminal Tower, Cleveland, Ohio
Bauhan, Mrs. George L. T., Welfare Council, 151 Fifth Avenue, New York City
Baumgartner, Rudolph, A. Iselin and Company, 36 Wall Street, New York City
Bell, Edward J., Jr., Department of Agricultural Economics, University of Wisconsin, Madison, Wis.
Benson, Edward G., Bond Department, National Park Bank, 214 Broadway, New York City
Beyleveld, A. J., Division of Agricultural Economics, Union Buildings, Pretoria, South Africa
Black, Stanley R., American Alliance Management Company of New York, Inc., 135 William Street, New York City
Bliss, Chester I., United States Department of Agriculture, 2224 Gentilly Road, New Orleans, La.
Bradley, James E., International Telephone and Telegraph Company, 67 Broad Street, New York City

Bragman, Charles, New York University, New York City
Breaznell, Eugene, National City Bank of New York, 55 Wall Street, New York City
Bredin, James H., Columbia University, New York City
Bristol, Lillian R., Index Number Institute, 349 Elm Street, New Haven, Conn.
Brittain, Alfred C., United States Department of Agriculture, Des Moines, Iowa
Brown, Maurice S., University of Michigan, Ann Arbor, Mich.
Butler, Albert, School of Commerce, University of Denver, Denver, Colo.
Callison, I. P., Wholesale Dealer in Crude Drugs, Union, Washington.
Campbell, W. W., General Electric Company, 530 Hanna Building, Cleveland, Ohio
Chalmers, Floyd S., Managing Editor, *The Financial Post*, Toronto, Canada
Chapman, John W., John Nickerson and Company, Inc., 61 Broadway, New York City
Cheney, Harriet M., Bureau of the Census, Washington, D. C.
Cocheu, Lincoln C., Jr., University of Michigan, Ann Arbor, Mich.
Cohen, Miriam F., Jewish Social Service Association, 71 West 47th Street, New York City
Connors, Myles F., L. F. Rothschild and Company, 120 Broadway, New York City
Corcoran, William M., 165 Broadway, New York City
Cowden, Dudley J., Department of Economics, Lafayette College, Easton, Pa.
Cullum, James B., Jr., J. H. Holmes and Company, Union Bank Building, Pittsburgh, Pa.
DeBard, August A., Jr., Chase National Bank, 20 Pine Street, New York City
Demond, Helen F., Department of Economics and Sociology, Mount Holyoke College, South Hadley, Mass.
Dunlap, Jack W., Territorial Normal and Training School, Honolulu, Hawaii
Epifanovich, Peter S., Apt. 10, 195 Claremont Avenue, New York City
Evans, Graham, Daniels and Fishers Stores Company, Denver, Colo.
Farra, Kathryn, Welfare Council, 151 Fifth Avenue, New York City
Farrington, Margaret, Department of Economic Research, H. L. Doherty and Company, 60 Wall Street, New York City
Friedman, Lloyd K., Department of Mathematics, University of Michigan, Ann Arbor, Mich.
Gaumnitz, Erwin A., Truman Furniture Company, 2600 Delaware Street, S. E., Minneapolis, Minn.
Ginsberg, William X., 179 Myrtle Avenue, Jersey City, N. J.
Goebel, Erwin J., Chicago, Duluth and Georgian Bay Transit Company, 110 West Adams Street, Chicago, Ill.
Grogan, John A., The Equitable Trust Company, 11 Broad Street, New York City
Hageman, Howard C., J. Henry Schroder Banking Corporation, 46 William Street, New York City
Hatton, Richard D., 714 Michigan Theater Building, Detroit, Mich.
Hawley, J. M., Jr., 2130 Paradise Street, Vernon, Texas
Haynes, Leo C., Bureau of Business Research, University of Texas, Austin, Texas
Hemala, Rudolph W., Western Electric Company, 195 Broadway, New York City
Herring, John P., 108 East 17th Street, New York City
Hill, Daniel L., University of Michigan, Ann Arbor, Mich.
Hillebrand, Henry, Metropolitan Life Insurance Company, 1 Madison Avenue, New York City
Hinrichs, A. Ford, Director, Bureau of Business Research, Brown University, Providence, R. I.

Humphries, Roger N., Market Research Department, Procter and Gamble Company, 320 Gwynne Building, Cincinnati, Ohio

Hutcheson, Mrs. Nancy M., Reports Department, Federal Reserve Bank, 33 Liberty Street, New York City

Jarvis, N. Leonard, Statistical Department, McClure, Jones and Company, 115 Broadway, New York City

Kailin, Harvey, University of Wisconsin, Madison, Wis.

Kelsey, Raymond T., Paine Webber and Company, 1111 Terminal Tower, Cleveland, Ohio

King, Fred E., University of Wisconsin, Madison, Wis.

Kripke, Albert E., University of Michigan, Ann Arbor, Mich.

LaMotte, L. H., Jr., Tabulating Machine Company, 50 Broad Street, New York City

Lansill, John S., President, United States Shares Corporation, 50 Broadway, New York City

Leager, Professor Marc C., Department of Statistics, North Carolina State College, Raleigh, N. C.

Leith, John D., University of Michigan, Ann Arbor, Mich.

Lipcovitz, Morris F., Department of Mathematics, University of Michigan, Ann Arbor, Mich.

Longley, Richmond W., Harvard University, Cambridge, Mass.

Louden, Clarke B., Statistical Division, F. W. Dodge Corporation, 119 West 40th Street, New York City

Lutz, Lauren J., University of Michigan, Ann Arbor, Mich.

Lynsky, Myer, 157 Callendar Street, Dorchester, Mass.

MacPhail, Douglas F., American Paper and Pulp Association, 18 East 41st Street, New York City

McClean, George A., Department of Economics, Standard Oil Company of California, San Francisco, Calif.

McCorkle, H. Robert, University of Michigan, Ann Arbor, Mich.

McCullough, Robert L., Investment Banking, National City Company, 55 Wall Street, New York City

Manning, Frances C., National Bank of Commerce, 31 Nassau Street, New York City

Mehlman, Charles, Department of Mathematics, University of Michigan, Ann Arbor, Mich.

Menge, Carlton H., University of Michigan, Ann Arbor, Mich.

Miklos, Madeleine E., 644 Riverside Drive, New York City

Mohler, Henry C., Department of Economics, University of Wisconsin, Madison, Wis.

Moore, Paul C., University of Michigan, Ann Arbor, Mich.

Murphy, Edmund J., American Electric Railway Association, 292 Madison Avenue. New York City

Neale, Walter G., Manager, Statistical Research Department, Wertheim and Company, 57 William Street, New York City

Norton, Clark B., Division of Research and Statistics, Federal Reserve Bank of Chicago, Chicago, Ill.

Paris, James D., Columbia University, New York City

Peel, Henry, University of Florida, Gainesville, Fla.

Perryman, Francis S., Royal Indemnity Company, 150 William Street, New York City

Pike, Sumner T., Case Pomeroy and Company, 60 Beaver Street, New York City
Pritzkow, Elizabeth, Bell Telephone Laboratories, 463 West Street, New York City
Pullar, Robert T., Columbia University, New York City
Quint, Julian V., Metropolitan Life Insurance Company, 25 Madison Avenue, New York City
Reaske, Gertrude C., Connecticut College, New London, Conn.
Reynolds, Merrill K., University of Michigan, Ann Arbor, Mich.
Rhodes, Thomas B., Division of Research and Statistics, Federal Reserve Board, 810 18th Street, N. W., Washington, D. C.
Rohr, Carl J., University of Denver, Denver, Colo.
Rudolph, Walter M., University of Southern California, Los Angeles, Calif.
Sayre, Robert A., American Telephone and Telegraph Company, 195 Broadway, New York City
Schneider, I. M., College of the City of New York, New York City
Schott, H. F., Orange County Children's Health Camp, Orange, Calif.
Schwartz, Kenneth C., American Alliance Management Company of New York, Inc., 135 William Street, New York City
Seelye, Seth H., Farmers Loan and Trust Company, 22 William Street, New York City
Sellery, George B., University of Wisconsin, Madison, Wis.
Shook, Bernice L., University of Michigan, Ann Arbor, Mich.
Simon, Sidney S., University of Michigan, Ann Arbor, Mich.
Smith, Harold W., University of Michigan, Ann Arbor, Mich.
Snook, Homer B., Victor Talking Machine Company, Camden, N. J.
Springer, William M., Bristol-Myers Company, Long Avenue, Hillside, N. J.
Stein, Harold, Department of Economics, College of the City of New York, 80 Willoughby Street, Brooklyn, N. Y.
Stephen, Edward A., The Ohio Bell Telephone Company, 750 Huron Road, Cleveland, Ohio
Strom, Andrew D., University of Michigan, Ann Arbor, Mich.
Thompson, Luther G., University of Michigan, Ann Arbor, Mich.
Timoshok, Igor P., Columbia University, New York City
Van Fleet, George S., Oregon State College, Corvallis, Oregon
Watson, Merrill A., Leather Company, 17 East Street, Boston, Mass.
Webster, Robert G., Eberle Economic Service, Inc., Los Angeles, Calif.
Weiler, Charles F., University of Wisconsin, Madison, Wis.
Wells, Oris V., United States Bureau of Agricultural Economics, Washington, D. C.
White, Warner G., White and Getman, Civil Engineers, 1103 Walbridge Building, Buffalo, N. Y.
Whiting, Douglas B., University of Michigan, Ann Arbor, Mich.
Whittemore, Eugene B., Rumford Press, 10 Ferry Street, Concord, N. H.
Whittington, Loren M., New York Telephone Company, 140 West Street, New York City
Young, Virginia J., New Jersey College for Women, New Brunswick, N. J.

REVIEWS

Statistical Laws of Demand and Supply, by Henry Schultz. Chicago: University of Chicago Press. 1928. 228 pp.

This book deserves careful study by both economists and statisticians, for it makes an interesting statistical approach to an important economic problem. The problem is that of finding for a given commodity the laws of demand and supply, and hence the elasticities of demand and supply, and the solution involves the ingenious application of correlation, curve fitting, and other statistical devices to price, consumption, and output data for sugar, the commodity chosen to illustrate the method.

The first and fourth chapters have to do with the theoretical considerations relating to the problem, those relating to demand in Chapter I and those relating to supply in Chapter IV. The nature of neo-classical, mathematical, and statistical demand and supply curves is discussed, and the meaning to be attached to the terms "elasticity of demand" and "elasticity of supply" is explained. Considerable space is devoted to the distinction between arc elasticity and point elasticity, and it is shown how the difficulties arising from the use of percentages in determining elasticity may in some cases be removed by the use of logarithms. Careful consideration is given to the opinions of other students of the subject—Cournot, Marshall, Moore, Davenport, Wicksteed, Auspitz, Lieben, and others—and to the different types of demand and supply curves which these authorities have set forth and discussed. In fact, if the book were useful for no other purpose, it would be valuable as a compendium of information setting forth what has been achieved in advancing our knowledge of the subject treated, up to the time the book was written. Finally, it is shown that the demand curve (or the supply curve) may be written in the form of an equation $x_0 = F(x_1, x_2, x_3 \ldots x_n, t)$ in which x_0 and x_1 are, respectively, the price and consumption (or output) of the commodity for which the elasticity of demand (or supply) is sought, $x_2, x_3, \ldots x_n$, the prices of other commodities, and t stands for time. If by some method, say, by partial correlation, the influence of the factors x_2, $x_3 \ldots x_n$ and t could be eliminated, we should have left the data in such form as to show the changes in consumption or output corresponding with a change in price, and hence obtain the elasticity of demand or supply.

In Chapters II and III the "law of demand" for sugar, as expressed by a statistical demand curve, is discovered, and from it is computed the elasticity of demand. The results obtained by several independent methods harmonize well, all approximating η (the elasticity of demand) $= -0.5$. Chapter V attempts for supply what Chapters II and III attempt for demand. The difficulties encountered, however, are much greater, and several values of e (the elasticity of supply) are found, depending on the data selected. For example, the elasticity of domestic supply is found to be somewhere between $e = +0.62$ and $e = +0.95$, while the elasticity of world supply is found to be $e = +0.57$.

In Chapter VI application is made of the results of the preceding chapters to the problem of computing the effects of a duty on the price, imports, domestic production, and domestic consumption of the taxed article. To this end use is made of the formula,

$$\Delta P = T_f \frac{1}{1 + \frac{e_d o_d - \eta_d c_d}{e_f o_f - \eta_f c_f}}$$

in which ΔP is the increase in domestic price; T_f, the duty; e, elasticity of supply; η, elasticity of demand; and d and f, "domestic" and "foreign" respectively.

As has just been noted, Dr. Schultz in his theoretic discussion suggested that the demand and supply curves might be derived from manipulation of the data by aid of the formula $x_0 \quad F\ (x_1, x_2, x_3, \ldots x_n, t)$, and devotes several pages (168–186) to an attack on the problem by applying to this formula the methods of partial and multiple correlation. Chief attention, however, is given to two other methods which he calls the method of relative changes and the method of trend ratios.

By the method of relative changes price-consumption or price-output data for the commodity studied are tabulated, each item is divided by the preceding item, and the link relatives so obtained are then plotted, the consumption or output relatives as abscissas and the price relatives as ordinates. A line fitted to the scatter so obtained is stated to represent the "law of demand" or the "law of supply" according as the data are chosen, that is, according as consumption or output is used and as contemporary or "lagged" prices are associated with the figures for quantity. Dr. Schultz does not always "lag" his data in getting a supply curve. He says (pages 137–138): "In short, the 'lag' oracle is not infallible; when the necessary conditions (assumptions) are not realized, it will sometimes be dumb, sometimes ambiguous, and sometimes lie." The method of relative changes is defended on the ground (quoted from Moore) that "by taking the relative change in the amount of the commodity that is demanded, instead of the absolute quantities, the effects of increasing population are approximately eliminated, and by taking the relative change in the corresponding prices instead of the corresponding absolute prices, the errors due to a fluctuating general price level are partially removed."

In the method of trend ratios it is assumed that points on a secular trend fitted to price-time data represent "normal" prices throughout the period covered by the observations, and that the corresponding points on a secular trend fitted to consumption-time data represent the "normal" quantities taken at the corresponding "normal" prices. We now compute the ratio of the actual price for each year to the "normal" price and, similarly, the ratio of the actual quantities taken to the "normal" quantities taken. A line fitted to the scatter plotted from the price-consumption ratios so obtained (consumption ratios as abscissas and price ratios as ordinates) is again stated to represent the law of demand or to be the demand curve. With a proper change in the data selected, the supply curve may be similarly constructed.

A matter which it is of great importance to note is this: *the test of confidence to*

be placed in the results obtained by either method is to be found in the correlation between the two sets of manipulated data. If the correlation is high, the result may be accepted with confidence; if low, the result must be rejected. The correlation, when using the method of relative changes, is computed between the price link relatives and the consumption link relatives. When using the method of trend ratios, it is computed between the price trend ratios and the consumption trend ratios. The data used for obtaining the link relatives or the trend ratios may be either adjusted or unadjusted. By adjusted data is meant "real prices" instead of money prices, and per capita consumption instead of total consumption. Certain theoretic reasons are advanced for preferring the adjusted data, but, after all, the real test of confidence in the results obtained is the test of correlation. If the correlation is higher with the unadjusted data, the result presumably would be held to be more trustworthy.

Having obtained the demand and supply curves by whichever method, the elasticities of demand and supply may be easily ascertained by applying to the curves the formula, $\eta = -\frac{dx}{dy} \cdot \frac{y}{x}$[1] and $e = \frac{dx}{dy} \cdot \frac{y}{x}$.

In estimating the value of Dr. Schultz's contribution to the subject, we must be on our guard against imputing to him a claim for having made discoveries which he does not in fact claim to have made. He has not extricated from the data demand and supply curves in the sense in which these curves are ordinarily defined in textbooks of economics, nor does he claim so to have done. On page 94 we read: "The law of demand derived in this study is a *dynamic* law: it describes in summary form the 'routine of change' of an important economic phenomenon. . . . It is quite different from the static law of demand of the classical writers. The statical law may be only approached but never realized in inductive investigation"; and again on page 189 the same statement in almost the same language is made with respect to his law of supply.

Nevertheless, his "law of demand" and his "law of supply" are expressed by curves, and these curves, as, for example, on pages 37, 59, 73, 83, 144, 158, are referred to as demand and supply curves; and, furthermore, from them are derived what he calls the elasticities of demand and supply. Since his demand and supply curves are not the "neo-classical" demand and supply curves, it follows that the elasticities derived from them are not the elasticities of demand and supply of the neo-classicists. They may bear no resemblance to them. Hence the need of caution. In applying the results of his method of analysis to any concrete problem, we must decide whether the problem calls for the curves and elasticities of the neo-classicists or whether those of his type are admissible.

Let us see just what the difference is.

The demand curve of the neo-classicists is assumed to be so constructed that at the prices of a given commodity represented by random ordinates, the quantities taken will be represented by the corresponding abscissas. It is further assumed, on the basis of human psychology, that a decrease in price will be ac-

[1] Dr. Schultz does not follow Marshall in using the minus sign. He says: "The demand equation will give the proper sign upon differentiation."

companied by an increase in takings, and, hence, that the curve descends to the right. The prices and the corresponding takings are also assumed to represent, as it were, a series of different observations all made at one and the same instant of time and in the same market, and, finally, during the experiment the prices of all other commodities in the market are supposed to remain constant.

The above description will apply also to the supply curve, if we substitute for the takings of buyers, the offerings of sellers, and note that, since a higher price will call forth greater offerings, the curve instead of descending will ascend to the right.

Obviously, as so defined, both curves are and must remain purely mental concepts. They have reference to a static society which has no real existence; but even if a static society did exist, it would afford no concrete evidence of the existence of these curves. For the price and the corresponding takings or offerings are determined by the point of intersection of the two curves, and if neither of the curves ever moved (as they would not in a static society), these data would never change. There would be one price and one quantity forever, which, plotted, would yield a single point—clearly inadequate data from which to construct a curve! The data which we do have, and which affords the only statistical evidence available of the existence of these curves, may, however, be conceived of as arising from motions of one or both of the curves. It is from such data that Dr. Schultz constructs his statistical laws of demand and supply. Indeed, that the demand curve (and presumably the supply curve) moves is recognized by Dr. Schultz himself. On page 27 we read: "But in order to bring the theory of consumption into closer agreement with the facts of our experience it is necessary to show how the demand curve *moves*—how it changes its position from time to time."

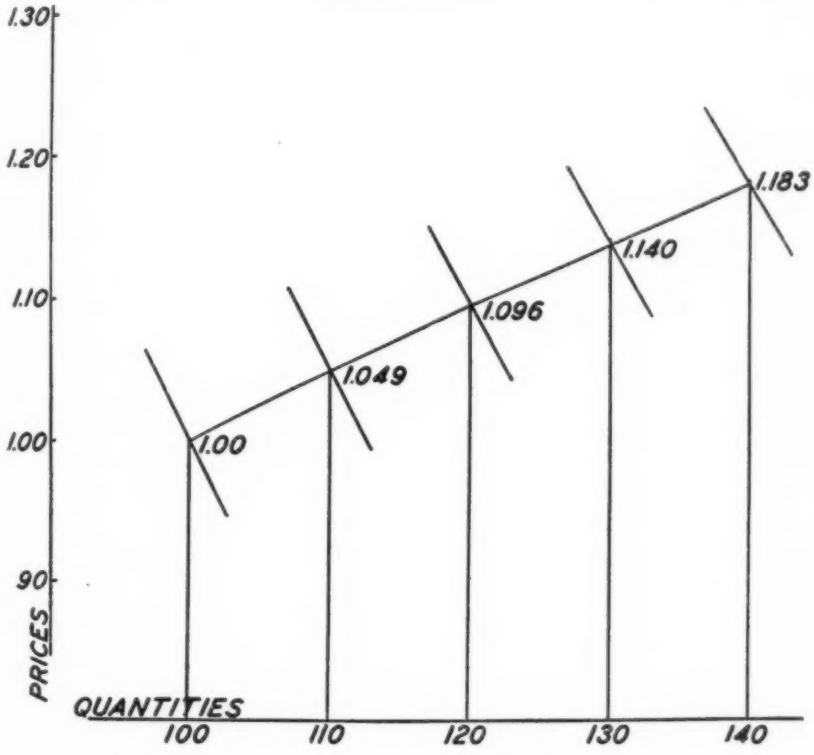
It may easily be shown that his curves bear no necessary resemblance to the ideal curves just described. We can show this by starting at the other end of the problem. Assume that we know the curves, construct data from their movements and then see whether from the data so obtained we can reconstruct the curves by applying Dr. Schultz's methods. A few simple illustrations of this process follow, in all of which a closed economy is assumed so that consumption and output may be regarded as identical.

First let us assume (Chart I) that e (the elasticity of supply) $=2$ and that throughout a series of five observations the supply curve remains constant, but that the demand curve, whose elasticity, η, is $-\frac{1}{2}$, moves steadily to the right, so that the outputs corresponding with its successive positions are 100, 110, 120, 130, and 140. We will assume that both supply and demand are represented by "constant elasticity" curves. The equation of the supply curve will then take the form $y^2 = cx$ and that of the demand curve $x\sqrt{y} = c$. Finally we will assume that the initial price when the output is 100 is \$1.00. The equation of the supply curve then becomes $y^2 = \frac{x}{100}$. From this equation the prices corresponding with the other assumed outputs may be computed. They are, respectively, \$1.05, \$1.10, \$1.14, and \$1.18. These conditions are shown in Chart I. The short lines crossing the supply curve are successive positions of the demand curve. Although

the curve changes its position, it may be assumed to retain the same elasticity ($-\frac{1}{2}$) in all its positions, and the short lines are so drawn as to have the slope of tangents at the points of intersection of the curves.

CHART I

SUPPLY CONDITIONS STATIONARY; DEMAND CONDITIONS CHANGE; DATA REVEAL SUPPLY CURVE

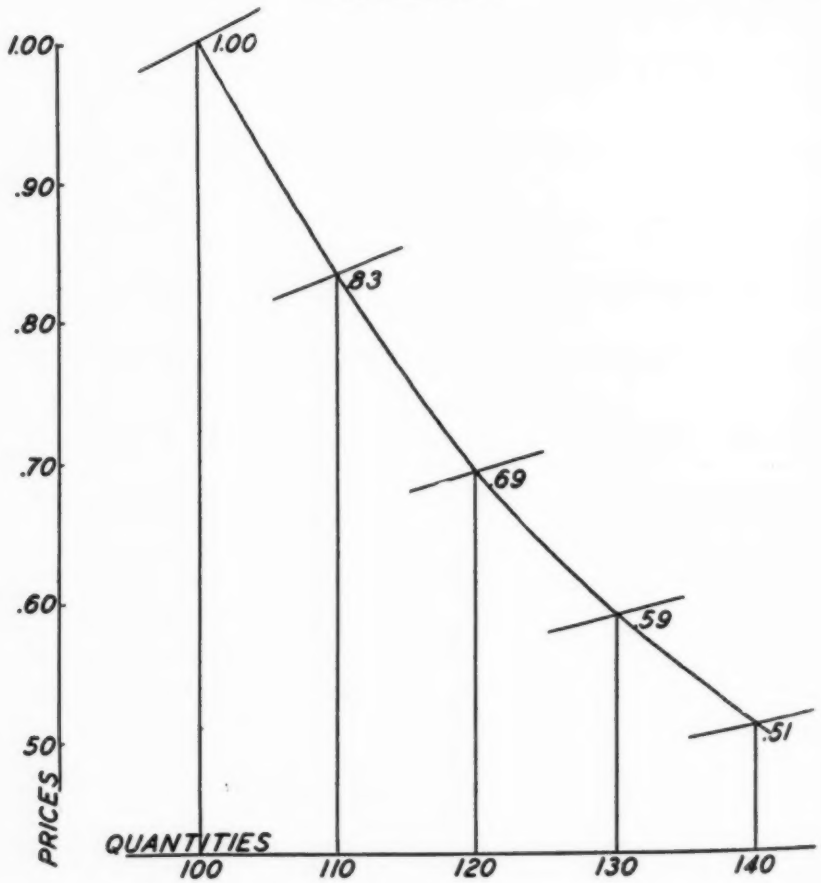


If, now, to the price output data we attempt to apply either the method of link relatives or the method of trend ratios, we are at once confronted with a difficulty: neither method will yield an appreciable scatter to which a curve can be fitted. Dr. Schultz, however, suggests (page 32) that percentage changes may be used instead of link relatives. If we compare the percentage changes in output with the percentage changes in price, we obtain for all the observations approximately 2—the assumed elasticity of supply. Thus, comparing the percentage change for the first two observations, we obtain $\frac{10}{100} \div \frac{.05}{1.00} = 2$. Making a similar comparison for the last two observations we obtain $\frac{10}{130} \div \frac{.04}{1.14} = 2.2$. No manipulation of the figures will yield the elasticity of demand.

Chart II is constructed on the assumption that starting with the same initial assumptions in regard to output and price the demand curve remains constant throughout the five observations, but that the supply curve, while retaining its original elasticity, moves progressively to the right. We obtain from the equation of the demand curve ($x\sqrt{y}=100$) as prices corresponding with the assumed

CHART II

DEMAND CONDITIONS STATIONARY; SUPPLY CONDITIONS CHANGE; DATA REVEAL DEMAND CURVE

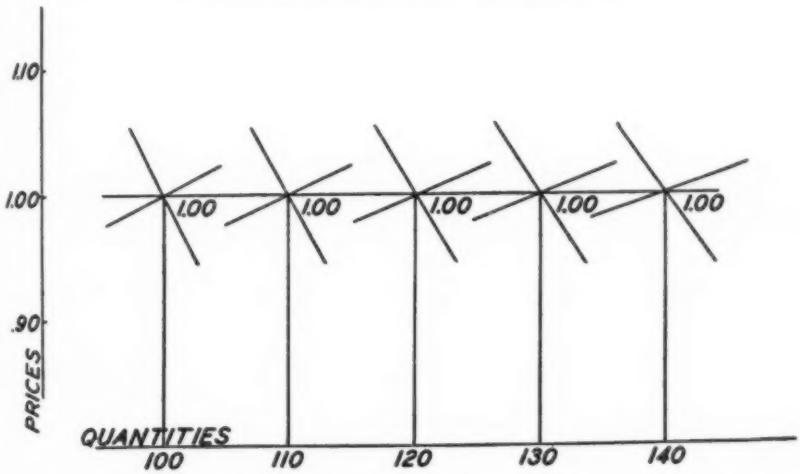


outputs, \$1.00, \$.83, \$.69, \$59, and \$.51 respectively. If we attempt to apply the method of link relatives or of trend ratios, we encounter the same difficulties as before, but by comparing percentage changes we may obtain a fairly close approximation to the elasticity of demand. Thus, comparing the percentage changes for the last two observations, we have $-\frac{10}{130}\div\frac{8}{59}=-.57$. If we had used

the last two observations going backward instead of forward, we should have had $-\frac{10}{140} \div \frac{8}{51} = -.45$. The average is very close to -0.5, the assumed elasticity of demand. The reason why we do not get the elasticity precisely is obviously because in using finite increments the elasticities obtained are "arc" elasticities rather than the "point" elasticities assumed in constructing the equations. No manipulation of the figures will yield the elasticity of supply.

CHART III

DEMAND AND SUPPLY CONDITIONS BOTH CHANGE; DATA AFFORD NO EVIDENCE OF EITHER SUPPLY OR DEMAND CURVE



Finally, we may assume that both curves move, as in Chart III, and so move as to maintain a constant price with changing outputs. In this case no application of any of Dr. Schultz's methods will yield any approach to the assumed elasticity either of demand or of supply.

The cases chosen are, of course, extreme, but the method here used may be applied to any price-output or price-consumption data of which the elasticities of demand and supply are assumed to be known. It will always be possible to account for the data by movements of the assumed supply and demand curves. We may then attempt to reconstruct the supply and demand curves by using any of Dr. Schultz's methods. We may succeed, but if we do it will be by accident.[1]

It is the reviewer's opinion that the neo-classical curves and the corresponding elasticities can be approximated from data only when, either through intimate knowledge of the industry or by some statistical device, we can impute fixity to one of the curves while the other remains constant. In that case the data will reveal the curve that remains fixed. (See Charts I and II.) More commonly, conditions both of supply and of demand change, in which case no manipulation

[1] Except, perhaps, when we use the method of multiple and partial correlation.

of price-output or price-consumption data, *taken by themselves*, will, except by accident, yield either the supply or the demand curve.[1] (See Chart III.)

But because Dr. Schultz's methods will not educe from data the neo-classical demand and supply curves, it does not follow that no importance is to be attached to his work.[1] For certain purposes it is of very great importance. What all of his methods, boiled down, amount to is this: Whenever between price and consumption (or price and output) data, manipulated by any of the methods which he employs, a high correlation is found to exist, it may be assumed with some confidence that the dynamic forces affecting supply and demand conditions have operated with some uniformity or according to some law during the period under observation. Hence, the curves or the elasticities derived from them may be used, as Dr. Schultz successfully uses them, for discovering the unknown price corresponding with a given output or consumption, or the unknown consumption or output corresponding with a given price. Such a test applied to an observation made during the period covered by the data should yield a close approximation to the truth, the higher the correlation the closer the approximation. And to the extent that it may be assumed that the dynamic forces will continue to operate thereafter in the same manner as they have been operating during that period, the methods may be used with some confidence for forecasting.

The propriety of applying the elasticities obtained by his methods to the problem of estimating the effects of a tariff is more open to question. The formula, $\Delta P = \frac{1}{1 + \frac{e_d o_d - \eta_d c_d}{e_f o_f - \eta_f c_f}}$, is computed on the basis of elasticities derived from the neo-classical supply and demand curves. It shows what effect the tariff will have on price, if supply and demand conditions (that is, the neo-classical supply and demand curves) undergo no change other than such change as is produced by the tariff. Will the elasticities obtained by his methods, when substituted in the formula, approximate the actual effect of the tariff on price, when the tariff is merely one of the numerous dynamic forces whose interaction results in the observed prices and outputs? If they will attain this end, the importance of Dr. Schultz's contribution to economic science can hardly be overestimated. But on this score the reviewer is skeptical, and Dr. Schultz himself is modest in his claims. He says, page 190: "The object of this study has been to make an exploration into the difficult and interesting field of statistical economics, not to develop methods of forecasting the price or the production of sugar." This is said with reference to price and production when no new disturbing factor is introduced. The reviewer is of the opinion that the likelihood of error in forecasting the price and production which will follow the imposition of a duty is even greater. There have been several changes in the duty on sugar in the last fifty years. It would be interesting to apply the formula, using his elasticities in each case where the duty was changed, and observe how closely the computed price of sugar following the change approximated the actual price.

One final point deserves notice: Dr. Schultz shows at some length that as the

[1] Except, perhaps, when we use the method of multiple and partial correlation.

correlation approaches perfection the coefficients of elasticity of demand and supply tend to approach numerical equality, but with opposite signs. With the interpretation given by him to demand and supply curves, such a conclusion is easily demonstrable. It follows from correlating the same data, first without, and then with, a lag. To the coefficients of elasticity derived from the neo-classical supply and demand curves, obviously no such principle applies. Either may vary from 0 to ∞ quite independently of the other.

It should be emphasized that the comment made in this review is designed, not to discredit, but to clarify Dr. Schultz's work. His work is a valuable contribution to the subject and should be welcomed by economists and statisticians.

Dr. Schultz requests that the following Errata be printed in this review.

ERRATA

1. P. 21, line 18, for $4.00 read $3.00.
2. P. 25, footnote 28, line 3, for *Mathématique* read *Mathématiques*.
3. P. 43, footnote 7, third formula, *read*

$$S_N=[\tfrac{1}{2}(\sigma_x^2+\sigma_y^2)-\tfrac{1}{2}\sqrt{(\sigma_x^2-\sigma_y^2)^2+4r^2\sigma_x^2\sigma_y^2}]^{\frac{1}{2}}$$

4. P. 139, figure 29. The graph for world production is wrongly drawn. The production for 1901 is plotted against 1902 and the production for 1902 against 1903, etc. See Table III, page 220, for the correct figures.
5. P. 173, line 14, *read* "the B's" as "the β's."
6. P. 175, line 5, *read* "(σ_1)" as "(σ_1^2)."

PHILIP G. WRIGHT

The Brookings Institution

Korrelationsrechnung (*calculation of correlation*), by Franz Baur. Leipzig and Berlin: B. G. Teubner. 1928. 57 pp.

This little handbook is different from anything available in American statistical textbooks. It is devoted, as its name indicates, to a consideration of correlation method. While the symbols seem needlessly complicated to an American reader, the mathematical development is simple and the explanations very clear. The theory of correlation is related to the theory of probability, and regression lines and their significance are treated as well as the coefficient of correlation. Multiple correlation is touched on briefly, but there is no treatment of non-linear regressions.

The book is superior to many American texts in its discussion of the logic of correlation analysis and of the significance of the constants obtained. The use of correlation methods in economic problems involving trend lines is discussed, and the limitations necessary in interpreting the results for such cases are noted.

MORDECAI EZEKIEL

United States Department of Agriculture

Stabilization of Prices, by Joseph Stagg Lawrence. New York; The Macmillan Company. 1928. xix, 484 pp.

The leading feature of this book is an attack on the doctrine that it is possible for a central bank to regulate the price level through the control of credit and particularly through the discount rate.

The main argument is that the cost of interest and discount on borrowed money is shown by an examination of the returns of national banks for the years 1918–1926 to be an insignificant percentage of the amount of credit extended by them. Mr. Lawrence calculates that whereas the gross yield of the national banks' loans, discounts, and investments averaged 5.663 per cent, the item of interest and discount on borrowed money spread over the same total gives an average of .2435 per cent.

Mr. Lawrence estimates the average rate charged by the Federal Reserve banks at 4½ per cent, and he infers that an increase or a decrease of the rediscount rate by 4½ per cent would raise or lower the rate charged by member banks to customers by only .2435 per cent. He even gives a table (p. 312) showing what rate charged by member banks corresponds to various rediscount rates from 4½ per cent up to 94½ per cent. For example, in order that member banks may charge 8.096 per cent, the rediscount rate must be 49½ per cent.

It would be difficult to find a more egregious example of the misuse of averages. A banker, when considering what rate of interest to charge a customer, does not ask what the average cost of rediscounts is to the whole banking community. Nor does he even ask what has been the average cost of rediscounts to his own bank over a period of years. He is concerned with the probable outcome of that particular transaction. If he thinks it probable that the contemplated loan will involve him in getting paper rediscounted, the rediscount rate will be the principal factor in his calculations. Otherwise the rediscount rate will not concern him (except in so far as it may have affected the state of the market through other bankers).

When Mr. Lawrence is dealing with the open market operations of central banks, he recognizes that sales of securities can make the rediscount rate effective. "They enable the Federal Reserve banks to enforce the rediscount rate in the New York money market," and also "to exercise some influence on the rate charged to clients in centers where there is some semblance of a market." He does not think that they have accomplished anything "in smaller communities" or in "the South and West." Mr. Lawrence is led to these conclusions by a statistical examination of rates of interest charged to customers by member banks. The places where open market operations do not appear to affect rates are no doubt those where there is a wide spread between the rates charged by member banks and the rediscount rate. But it is not to be inferred that the action of the Federal Reserve banks is entirely without effect on credit in those places. The reluctance of many banks to borrow from the Federal Reserve banks is well known, and such banks, when driven to rediscount, will put pressure on their customers not to borrow.

In any case, in order to exercise control over credit, it is not necessary that a change in the rediscount rate should be passed on to *all* borrowers. It is quite

enough to reach some of them, provided those affected include an important enough group of borrowers sensitive to the rediscount rate.

With regard to the sensitiveness of borrowers, Mr. Lawrence argues (p. 325) that "interest charges on borrowed money constitute such a small part of the total expenses of the average enterprise that it is difficult to see how they can influence, substantially, the commitments which a business man is willing to make, provided other factors are favourable." Yet a moment before (p. 321) he has said that "we are likely to find many conservative business men operating on fractional percentage margins. There is every probability therefore that a rise of several per cent in the interest rate will bar marginal borrowers . . ." In attacking the principle of the susceptibility of credit to regulation by a central bank, he might have been expected to give this vitally important question of the sensitiveness of borrowers a more thorough treatment than in two meager and mutually contradictory passages.

The main part of the argument regarding the discount rate is directed to American conditions and American experience. One short chapter (pp. 412–431) deals with bank rate and price control in England, and includes some tables of French and German post-war statistics. For England a table is supplied collating annual averages of bank rate and the price level from 1845 to 1927. Mr. Lawrence supposes that according to the "bank rate theory of price control," an advance in the rediscount rate should be associated with a fall in the price level, and vice versa. But his table indicates the contrary: "Fifty-one changes in a total of 82 indicate similar discount rate and general price movements, while in only 23 cases is the orthodox theory supported" (pp. 421–426).

His interpretation of the data is completely vitiated by his disregard of the well-known principle that rising prices are associated with high short-term interest rates and falling prices with low rates, though he himself has enunciated the principle clearly enough on pages 322–323. ("Is an interest rate," he there asks . . ."a cause or an effect of price factors? It obviously cannot be both cause and effect.")

According to the bank rate theory of price control, a rise of the price level should be checked by raising the interest rate above the high rate corresponding to the rising prices, and a fall of the price level should be checked by lowering the interest rate below the low rate corresponding to the falling prices. No merely statistical analysis of a table of price levels and bank rates could possibly disentangle their interaction, for the interest rate *is* "both cause and effect."

For all his eighty-three years of statistics, Mr. Lawrence does not contend that "the discount rate as manipulated by the Bank of England has been entirely without effect" (pp. 426–427). But it is not easy to discover just what sort of effect he allows to it.

And is there not a danger that if he proved his case, he would prove too much? For how is the gold standard maintained? It is necessary to keep the value of the currency unit equal to the value of a prescribed quantity of gold. The essential condition is that the internal price level must be kept in equilibrium with the external. Mr. Lawrence shows clearly enough that a discrepancy between the internal and external purchasing power of the monetary unit cannot con-

tinue, but must be corrected by a movement of the foreign exchanges (pp. 23–24). But with a gold standard, the movement of the foreign exchanges is limited, and an inflow or an outflow of gold must occur, which cannot be checked *unless* the discrepancy between internal and external prices is corrected. If there were no central bank, the correction would have to be brought about by the change in the volume of currency itself (together with consequential changes in the credit situation). But where there is a central bank, the volume of currency is determined mainly not by the amount of gold in the country but by the lending policy of the central bank. The lending policy of the central bank (including its open market policy) is an essential link in the chain by which equilibrium is regained. If the central bank cannot cause a decline of internal prices through a restriction of credit, then this essential link is missing and the gold standard cannot be maintained.

Mr. Lawrence says that "the great majority" of stabilization plans "lean heavily upon the quantity theory" (p. 432) and that the quantity theory means that in the equation of exchange "the causal factor . . . is the quantity of money" (p. 435).

Here he is quite mistaken. The majority of the authors of these plans accept some form of the quantity theory, as does Mr. Lawrence himself. But even Professor Fisher, when he argued that except at times of transition the price level is never cause but always effect, did not say that the other factors (velocity and volume of transactions) might not be causal. Mr. Lawrence quotes the principle that "the changing value of money is itself a cause of a change in the velocity of circulation" (p. 452), and says that "Keynes himself" gives an illustration of the principle in his *Monetary Reform*. But, far from this being an admission by "Keynes himself," no one has laid more stress on this aspect of the quantity theory than Mr. Keynes. Professor Fisher is more closely identified with the advocacy of the quantity theory than any contemporary economist. Yet even his stabilization plan does not *depend* on the quantity theory. It depends, like the gold standard, on the possibility of keeping the currency unit equal in value to a prescribed amount of gold, and only differs from the gold standard in that the prescribed amount of gold would be subject to variation from month to month.

The quantity theory, like credit control, is no less applicable to the gold standard than to Professor Fisher's plan.

As a result of his preoccupation with the general question of credit control, Mr. Lawrence gives very inadequate treatment to the subject of price stabilization itself. In the introductory part of the book, he deals with the injurious consequences of instability, and he reaches the conclusion that "changes, particularly rapid changes, in the value of the monetary unit are viciously detrimental to the equitable distribution of individual group and national incomes" (p. 26). Not a word about unemployment! Even when he is insisting that "falling prices have a distinctly depressing effect on business" (p. 456), he mentions the depressing effect on profits but not on employment.

He avoids the difficult question of the exact meaning of changes in the value of the monetary unit, and has very little to say about index numbers. He urges as an objection to stabilization that "if the general price level remains

fixed, then every advance of a commodity or group of commodities *must* be compensated by the fall in price of some other commodity or group of commodities" (p. 461). But if he objects to the price level being fixed by reference to an index number, in what sense must it be fixed in order to avoid those changes which he regards as "viciously detrimental"?

The Genoa Resolutions are quoted (pp. 162–165), but Mr. Lawrence's only comment is that they require an international agreement, and he quotes Professor Taussig as writing in 1913 that an international agreement for the adoption of the Fisher plan seemed to him "in the highest degree unlikely." That is not very relevant to resolutions, embodying a *different* plan, which *have* been adopted by international agreement.

Mr. Lawrence's neglect of the international aspects of his subject is a serious defect. The abandonment of the gold standard is hardly a practical issue at the present time, either in America or in other gold standard countries. Consequently the questions of immediate interest are those relating to price stabilization with *retention* of the gold standard. The wealth value of gold is determined in world markets, and it is beyond the power of any one country to stabilize it without regard to the action of the others. Of the relation between the value of gold and the demand for it for monetary purposes in the gold standard countries of the world, as determined by their respective currency laws and practices, he has nothing to say (except that he recognizes the effect on the value of gold of the abandonment of silver currencies in favor of gold in the period 1873–1896—pp. 204–214).

In what is, no doubt, a well meant endeavor to enliven an austere subject, Mr. Lawrence has been led to adopt a literary style which occasionally becomes offensive. And he is none too careful to make sure that he understands the authors he quotes. For example, he says that Mr. Keynes's "reasoning at times lapses in its accuracy," and quotes as an instance from *Monetary Reform*, "If we glance at the course of prices during the last five years it is obvious that the United States, which has enjoyed a gold standard throughout, has suffered as severely as many other countries . . . " "It must be news," Mr. Lawrence comments, "to Americans familiar with the history of the past five or six years that this country has borne a crown of thorns in the form of the gold standard."

The "crown of thorns" is a characteristic example of Mr. Lawrence's habit of labelling his antagonists' views with rhetorical phrases which they have never used. The phrase entirely misrepresents Mr. Keynes's meaning, for his point is not that the gold standard caused the troubles, but that it did not prevent them. And perhaps it is pertinent to point out that, as *Monetary Reform* was written in 1923, "the last five years" were the years 1918–1923, not Mr. Lawrence's "past five or six years."

R. G. Hawtrey

The Construction of Index Numbers, by Warren Milton Persons. Cambridge: Houghton Mifflin Company. 1928. 90 pp.

Those of the statistical fraternity who, instead of working cross word puzzles, prefer to continue the quest for a perfect index number formula—a formula which, when applied to a given mass of data, will produce an index series having the same trend no matter what base is selected and no matter what interrelationships may exist between prices and quantities—a formula which will yield an index series representing the central tendency of a large group of series computed by the aid of various formulas—will find Dr. Persons' little book very helpful and illuminating. He takes up the problem where Professor Fisher drops it. Some of Professor Fisher's conclusions are criticized, but Dr. Persons devotes most of the early part of his book to showing more clearly how the behavior of various averages of relatives is affected by correlations existing between prices and quantities. Both the theoretical and the experimental work presented throw new light on this problem.

On page 2 we find the following statement: "The object of the present study is that of measuring the average change of prices of two or more commodities during two or more periods. In other words our problem is that of averaging a number of price relatives." Furthermore, on page 9 we find this sentence: "In searching for an answer to the question, 'What is the average change of prices between two periods?' we encountered the following difficulties." On page 32 the following statement occurs: "We may justly characterize the price index as logically superior to the other indexes that we have examined for a binary comparison."

These quotations indicate that Dr. Persons still believes that, for a given set of data, there is one average which, if it can be discovered, will give results superior to those obtained by any other average. To the reviewer, such an assumption seems wholly indefensible, for it implies either that some index series exists which will answer correctly all sorts of questions or that index numbers are not devices for answering questions but are ends in themselves—mathematical entities divorced from the world of economics and business. Apparently, beliefs in the existence of such phenomena as "base-bias" and in the efficacy of the "base reversal test" and the "circular test" have their origin in the latter assumption. Just why the index series occupying the central position in a group of index series has any claim to merit is not clear to the reviewer; nor does he understand why it is deemed desirable that when the index number is computed forward and backward a formula should give values which are reciprocals of each other. The fact must, however, be recognized that Dr. Persons' view of the subject corresponds with that of many of the most eminent students of index numbers.

In Section IV of the book, Dr. Persons discusses in a very clear way the difficulties of obtaining in the correct form the data desired for the construction of an index number. He calls attention to the fact that commodities are always changing in nature and that this fact makes accurate price comparisons between distant points of time impossible.

Section V is devoted to a discussion of the comparative behavior of chain and fixed-base indexes, and some of the relationships between these two groups are

clarified. Dr. Persons deserves credit for aiding in the solution of this complex problem.

The book shows evidence of careful preparation and sets forth the results of a large amount of painstaking research.

WILLFORD I. KING

New York University

Cours de Statistique, by Albert Aftalion, compiled by Jean Lhomme and Jean Priou. Paris: Les Presses Universitaires de France. 1928. 319 pp.

This book presents the elements of statistics as a method of characterizing relations among phenomena. The book consists of three main parts. The first part includes graphic representation, averages, index numbers, time series, and economic cycles. The second part deals with measures of correlation under the rather non-descriptive title "Les lois or les régularités statistiques." Correlation is discussed by the use of various coefficients including the Pearson correlation coefficient and by the regression method. The third main part presents the subject of forecasting with special reference to economic phenomena. The logical and technical difficulties of forecasting are discussed briefly. In this connection, human phenomena are contrasted with certain physical phenomena. In discussing the empirical technique and procedure in forecasting, the author draws very largely from American economists and statisticians.

The exposition is clear throughout and the illustrations are well chosen to interest students of economics. On the whole, it seems correct to say that the book is very similar to the considerable number of elementary books on statistics that have been published by American authors within the past few years.

H. L. RIETZ

The Prosperity of Australia, by Frederic C. Benham. London: P. S. King & Son, Ltd. 1928. x, 276 pp.

In this blend of qualitative and quantitative economic analysis, Mr. Benham moves through these three stages: (1) He develops measurements of the Australian national income and wealth from 1901 to 1926; (2) he indicates how these are affected by the protective tariff and state wage regulation, as well as by certain other factors; and (3) he makes some specific recommendations in the direction of lower tariff, less curbing of supply and demand in wages, and more social insurance accompanied by heavier direct progressive taxation. "I know only too well," he remarks in closing, "that, in Australia, my views will be unpopular"; and he gives particulars as to rebuffs received from the Tariff Board. It is clear enough that both business and labor interests will find parts of his message unwelcome, so that we must admire his courage in urging it upon them. Its scientific quality is high, even if but few direct references are made to scholarship outside Australia; and foreign economists will find the book a welcome introduction to Australian data.

The first chapters deal with measurements of the country's national income,

wealth, and other indicia of its "prosperity," which the author uses as approximately synonymous with economic welfare. The official publications of Australia are unusually full of materials for estimates of income. Besides the income-tax and the comprehensive labor statistics, there are *annual* censuses of manufactures, agriculture, and some other types of "production." Like Bowley and Stamp, Mr. Benham attempts to avoid counting public expenditures for the more than 70,000 war, old-age, and invalid pensioners, as well as payments on account of war loans, calling these mere "transfers" of income. Capital gains and business losses are excluded from the reckoning, while no allowance is made for evasion of income tax. The author attempts to include rental values of dwellings and all other fixed property, private and public; and he recognizes that income consumed directly by its producers (especially farmers) is probably underestimated. All this shows clear thinking as to principles. Numerous calculations, tables, and charts in the end purport to show that the "home-produced income" of Australia in 1924–25 was around £667,000,000. Interest, dividends, etc., due to foreign equity-owners are estimated at nearly £32,000,000 (£24,000,000 being interest on borrowings of the Commonwealth and state governments), leaving about £635,000,000 for the folks at home. Mr. J. T. Sutcliffe's estimate for 1920–21 is corrected by various additions and subtractions which nearly balance one another, so that Mr. Benham makes the "home-produced" income for this earlier year £532,000,000, with £25,000,000 due to foreign owners. The total population of Australia in 1924–25 was around 5,874,000, so that the national income was some £108 at current prices.

Further computations, utilizing several series of index-numbers, indicate that real income per head increased by 25 to 30 per cent from 1901 to 1909; remained about constant to 1913, then slumped 10 to 20 per cent during the war period, and had not quite regained the pre-war level by 1926. Apparently in this whole period the advance of income per capita was markedly less than in the United States. Estimates are cited on the national wealth and indebtedness for several points within the same period. According to government inventories as of 1921, the total wealth of Australia amounted to over £2,800,000,000—three fourths of it private and one fourth government and municipal property. These same proportions are thought to apply to Great Britain; while the Federal Trade Commission recently estimated that only 12 per cent of American national wealth belongs to federal, state, and local governments. Australian indebtedness to foreigners in 1921 is estimated by Mr. Benham at nearly £500,000,000—under 20 per cent of the total value. In contrast with more or less comparable estimates for the United States, the per capita wealth in Australia, deflated as to price by wholesale index numbers, showed a recession from 1901 to 1915 which had not been made up by 1925.

Nothing is said about the distribution of wealth among families, which is a bit curious in view of the apparent care with which Australian official statisticians work up annual figures of estates probated and taxed, in connection with total mortality. A brief section on distribution of income is featured mainly by Sutcliffe's table purporting to show less inequality than exists in America. An extremely interesting revelation of the population census of 1921 was that the average

number of children of employer-husbands was 3.55, of husbands working on their own account, 3.79, and of wage and salary-earners, 3.02. Presumably, differing age-distributions in these groups may have something to do with the case, but anyhow Mr. Benham's inference that people with lower incomes (wage-earners) may have somewhat fewer dependents, on the average, than those of higher incomes, seems well founded. In other connections (pages 79, 207) he makes the statement, apparently on the basis of some rather hypothetical calculations, that "ownership" takes only one quarter of Australia's income, instead of one third or more as in Great Britain and elsewhere.

Natural resources and protection are treated at some length, the conclusions being that Australia's resources per capita, on the whole, compare favorably with those of other countries, and that increasing protective tariffs have hindered rather than promoted prosperity. Then follow what are to me more intriguing chapters dealing with wage-regulation, admirably seasoned, like the earlier parts, with statistical procedures. The author shows, on the one hand, that the wage-regulating authorities, always purporting to be guided only by the needs of the employee, have in fact responded to changing situations of supply and demand. On the other hand, they appear to have produced some extra unemployment by keeping wages uneconomically high at times. In Table 39, for example, Mr. Benham uses census of production data to compare, year by year, the percentage which total wages constituted of the total sales value of the product with the percentage of workers in the industry unemployed during the year. The direct correlation is +.62. He thinks that differentials between skilled and unskilled have been driven too low; that average real wages, allowing for unemployment, have not increased more than other incomes; and that a national minimum income for each mouth to be fed is much more important than a minimum wage. He therefore recommends the nation-wide adoption of more (contributory) social insurance, including the "child endowment" scheme already in force in New South Wales. In this state the wage boards attempt to set the minimum wage according to the needs of man and wife only. The employer is also required to pay some 3 per cent of his wage-bill into a fund, out of which workmen of sufficiently low income are allowed to draw for the support of actually dependent children.

If Mr. Benham's views are unwelcome to politicians, most of them are congenial to moderately conservative economists. His critical treatment of two favorite Australian policies, protection and wage-regulation, gives, perhaps, some special weight to his explicit or implied approval of two other policies which the conservative in general regards with suspicion, namely, social insurance (particularly family allowances) and extended state industries. Nowhere do I find him prophesying that family allowances will produce overpopulation, or crying for "more business in government and less government in business."

Z. Clark Dickinson

University of Michigan

Business Cycles and Business Measurements: Studies in Quantitative Economics, by Carl Snyder. New York: The Macmillan Company. 1927. xvii, 327 pp.

This book should be very helpful to the business man eager to acquire some knowledge of recent statistical findings regarding the business cycle, as well as to the student who is receiving instruction of a more formal sort in classes. It is not, however, intended to form a complete and self-sufficient treatise on the entire business cycle problem.

The organization of the book is admirable. The text is clear and incisive, the 56 charts are thoroughly effective in themselves, and are adequately supported by first-rate descriptions of source, composition, weighting, etc., and by a 75-page Appendix of tables. A special and very useful feature is the alphabetically arranged index of charts and tables at the back of the book, where a short bibliography is also offered along with the usual index to text.

Naturally, Dr. Snyder's own work in conjunction with his staff at the New York Federal Reserve Bank figures largely in this group of "studies in quantitative economics." The author introduces the description of these studies with four short chapters. The first of these, "The Nature of Business Cycles," contains some A B C economics; the following three are devoted to some A B C statistics: the measurement of economic growth (including non-linear secular trends), the isolation of business-cycle forces from the cross-currents of other sorts affecting almost any time series of raw economic or business data, and, finally, that important but oft-neglected problem of choosing an appropriate base period for index numbers.

After this introductory matter, Dr. Snyder describes *in extenso* his weighted general index of the "Volume of Trade" and its five major constituents—Production (of producers' and consumers' goods separately), Primary Distribution, Distribution to Consumers, General Business Activity, and Financial Activity—and also treats at some length individual series comprising each of these major constituents. The chapters following contain a description, with some critique, of bank clearings (Chapter VI) and of the velocity of bank deposits (Chapter VII) as measures of business cycles. After hasty review of certain other measures of the business cycle, he discusses the difficult and moot question of applying price deflators to dollar value series. In this respect he goes further than almost any other writer has gone—further, indeed, than many would be willing to go—in accepting certain price indexes as proper deflators for certain dollar-volume series now available.

The author treats, I think, too lightly the difficulty of securing a price series truly reflecting the economic composition of a dollar-value total such as bank clearings or bank deposit turnover. His view may be summarized as follows (pages 180-181):

> It may be freely admitted that the method is not without its dangers, especially in inexperienced hands. But if the findings are checked, step by step, with every kind of quantity data available, as our knowledge widens we grow to confidence in the results. For the rest it seems to the writer that much of the distrust which many feel toward deflated series arises from two misconceptions. The first is that a meticulous accuracy is attainable even with our most cherished quantity data. It is not. And the other is that the "deflating"

indexes, to be of value, must themselves be of meticulous accuracy. For the most part price changes, and therefore these deflating indexes, are slow moving and do not often affect the cyclical swings. The huge upheaval of 1919–1921 was quite extraordinary, and, like that of our Civil War, may not recur in half a century. In periods of slowly changing prices, price changes in any series, to a large extent, will be taken care of in the computation of a long-time secular trend.

The chapter on business failures as a cyclical barometer adds little conviction as to the dubious merits of that variable. The chapters on prices, and especially that on interest rates, contain some thoughtful contributions which the student and the business man alike will value.

W. A. Berridge

Metropolitan Life Insurance Company

An Analysis of Bank Statistics for the United States, by Allyn A. Young. Cambridge: Harvard University Press. 1928. vii, 78 pp.

This book consists of a series of four articles reprinted from the *Review of Economic Statistics*, 1926 and 1927. It presents, in compact form and with "little or none of the interpretative analysis which the figures obviously invite," a number of time series of banking and monetary statistics, almost all of them compiled from the annual reports of the Comptroller of the Currency. The primary data cover an extended period—for some series nearly 60 years (1867–1926)—with figures for irregular intervals, determined by call dates, which approximate two or three months. For the period 1901–1926, for which the basic figures are subjected to refinement, an analysis of the data is made with a view to determining seasonal, cyclical, and regional characteristics.

Some of the principal inferences drawn from the data by Professor Young may be briefly enumerated as follows:

1. The combined loans and investments of banks, their total earning assets, have increased more rapidly than their invested capital.

2. Investments have increased more rapidly than loans, especially since 1888, and more especially for banks outside New York City. The banks generally increased their holdings of investments most rapidly when bond prices were relatively high, and vice versa. "The increase since 1915 of the amount of investment securities held by the commercial banks of the United States has had the proportions of a veritable revolution." In general, however, since 1915, the growth of investments has been much less rapid than the growth of time deposits.

3. "Money flows into hand-to-hand circulation in the early winter and in the spring, when retail trade is especially active; later in the winter, and in the summer, when retail trade is dull, money accumulates in the banks." The conjecture is advanced that the seasonal variations of the money in merchants' tills count for more (except probably at harvest time) than the seasonal variations of money in peoples' pockets.

4. The amount of money in hand-to-hand circulation (*i. e.*, outside banks) increases and decreases from time to time for reasons other than seasonal; these

"cyclical fluctuations" generally lag behind those of wholesale prices, frequently by a substantial interval. Since the establishment of the Federal Reserve system, but not before, seasonal changes in the amount of vault cash held by the commercial banks have correlated directly with changes in the amount of money in circulation.

5. Deposit currency—defined as deposits exclusive of interbank deposits—showed, prior to 1915, considerable seasonal elasticity, at least for the country as a whole.

6. For the period 1915–1926, the magnitude of seasonal variations in loans, deposits, etc., was certainly not smaller, and in some instances was larger, than before the Federal Reserve system was established—but the nature of the seasonal fluctuations was different.

7. "I see no basis for the belief that these cyclical swings (of industry, prices, trade), once under way, were never halted until the resources of the banks had been exhausted. Monetary factors . . . set limits . . . but only in cycles of exceptional magnitude do such limits become effective."

8. Deposit currency and money in circulation (money outside of banks) do *not*, since 1915, expand and contract at the same rate; *i. e.*, the ratio of the one to the other is far from constant. This point is commended by the author "to students of monetary theory, and particularly to those who have been accustomed to assume that the ratio is constant and that variations of the *sum* of bank deposits and money in circulation are sufficient to explain all the changes, even short time changes, of the general level of prices"; *variations* in the ratio are essential parts of the "cycle" mechanism.

A distinguishing characteristic of the study, and one that is stressed, is that almost all of the series are given separately for New York City banks and banks outside New York City—not for all national banks combined. It is not altogether clear, however, that this procedure is uniformly fruitful, although some interesting contrasts are developed: At one season (summer to autumn) bank holdings of vault cash went down in the interior and up at New York; at another season (early autumn to late autumn) deposits went down at New York and up in the interior. The second of these relationships is attributed in part, and quite justly, to "changes in the direction of the net balance of payments." "Money flows back to the interior"—so runs one qualified statement—"because loans and investments have expanded there. Loans and discounts expand in New York because money has flowed there." The reverse of this proposition, it seems to the reviewer, could be quite as well substantiated; the proposition itself, perhaps, would have been qualified further if the study had recognized, as it nowhere seems to do, the fact that the loans of outside banks, as reported to the Comptroller, have always *included* their street loans made in New York City. One may hesitate, furthermore, to conclude from the data, as Professor Young seems to do, that the causal relation, if any, between fluctuations of the banking series and "business activity" is probably closer for the "outside" series.

A very salient part of the study undertakes the regional dissection of the data, in respect both to seasonal and to cyclical aspects. The fact is brought out, for instance, that in the south and west bank deposits decrease seasonally in the

spring and increase seasonally in the fall, while in the eastern and middle western states their seasonal movement is exactly the opposite of this; this points to the special significance of "seasonal changes in the balance of payments between a section and the rest of the country." It is also indicated, as an inference from the diversity of the regional series, "that geography has something to say on the negative side of the question of the significance of the notion of a 'typical' cycle."

The compilation of the basic series that are here brought together is of distinct value to students of banking developments. They obviously invite, furthermore, the very type of "interpretative analysis" that this study purports to extend but sparingly—as by comparing them with series representing the course of production, employment, prices, etc. Certain inherent shortcomings of the data, however, set distinct limits to their value. Their merit resides chiefly in their length—about 60 years; they are not only not on a monthly basis but they suffer quite evidently from their origin—in reports devised by, or at least primarily for the use of, authorities charged with enforcing specific laws and regulations upon individual banks. They are essentially statistical by-products of the function of "examining banks"; that they can in fact be used to illuminate the course of economic history, but somewhat dimly, is almost accidental. The problem really requires data collected more nearly *ad hoc*, like the new but lengthening series, daily, weekly, or monthly, for brokers' loans at New York City, bankers' acceptances outstanding and held by banks, commercial paper outstanding, money in circulation, transfers from section to section through the gold settlement fund of the Federal Reserve system, and more discriminating and comprehensive regional data of various kinds, including bank deposits subject to check and classified loans of all classes of banks.

CARL E. PARRY

Federal Reserve Board

Stock Dividends: Report of the Federal Trade Commission made in response to Senate Resolution No. 304, 69th Congress, 2d Session, approved December 22, 1926. (Senate Document No. 26, 70th Congress, First Session.) 273 pp.

This is an exhaustive report of the results of a questionnaire sent to some 18,000 corporations, with primary reference to the amounts of stock dividends issued during the years before and after the decision of the Supreme Court of the United States (Eisner *v.* Macomber, 252, U. S. 189, March 8, 1920) which held that stock dividends were not taxable as income.

The purpose and plan of the investigation can best be shown by reference to the terms of the Senate Resolution which authorized the inquiry:

> WHEREAS it has become the usual practice of corporations in order to protect stockholders from the payment of income taxes, to declare stock dividends; and
> WHEREAS this procedure enables corporations to acquire competing plants, and in this way avoid the provisions of the anti-trust laws; and
> WHEREAS in order to legislate upon the subject, the Senate should be fully informed as to the extent of this practice: Therefore be it
> *Resolved*, That the Federal Trade Commission be, and it is hereby, directed to ascertain and report to the Senate the names and the capitalization of corporations that have issued stock dividends, together with the amount of such stock

dividends, since the decision of the Supreme Court holding that stock dividends were not taxable, and to ascertain and report the same information as to the same corporations for the same period of time prior to such decision.

The statistical work of the Commission was planned intelligently, considering the limitations imposed by the Senate Resolution, and the results have been compiled with skill. It was apparent at the outset that a report compiled in literal compliance with the terms of the Senate Resolution would be very misleading on account of the statistical bias which would result from tabulating only companies which issued stock dividends after the decision. Accordingly, information was asked from all corporations known to have issued stock dividends at any time from 1913 to 1926. The questionnaire called for the amounts of stock dividends issued, the amount of stock outstanding, and the total surplus and undivided profits shown by the books at the closing date nearest the end of each year from 1912 to 1927 inclusive. Reports were obtained from 10,245 corporations which issued stock dividends during the years 1920–1926, but from only 303 more which issued stock dividends in 1913–1919. For these 10,245 corporations the report gives the names and the information pertaining to them which was called for by the Senate Resolution, and also a summary. There is also an analysis of the combined records of 2,971 corporations from which reports on dividends and stock capitalization were obtained for the whole 14-year period, and another summary of the records of 1,000 corporations shown by the financial manuals to have issued stock dividends at some time during the 14-year period. These tabulations make obvious the effect of the taxation of 1920 on the rate of issuance of stock dividends. For the 2,971 corporations the amount of stock dividends issued from 1920 to 1926 was 476 per cent greater than in the years from 1913 to 1919, while the amount of cash dividends was only 73 per cent greater. For the sample of 1,000 corporations, the increase in cash dividends was again 73 per cent, while the increase in stock dividends was 439 per cent.

The Commission is very cautious in the interpretation of its data. There is an excellent discussion of the factors which contributed to the reinvestment of profits in the war and post-war period and of the effect of taxation in encouraging reinvestment of earnings while discouraging the capitalization of surplus.

While there is no direct comment on the economics of the Resolution, the discussion makes obvious the error in the first paragraph of the Senate Resolution. It is not the declaration of stock dividends, but the failure to declare cash dividends which protects stockholders from the income tax. There is no reference to the still more preposterous doctrine of the second paragraph, namely, that the declaration of stock dividends enables corporations to acquire competing plants and thus evade the anti-trust law.

The body of the report closes with the cautious generalization that "even though it is necessary or desirable for one reason or another to pursue a policy of building up a large surplus from earnings and reinvesting it in the business, it does not follow that it is either necessary or desirable to capitalize that surplus to the extent prevailing in the last few years." This is true. So far as the reviewer can see, however, it would be equally true to say that "it does not

follow from anything in this report that it is *undesirable* to capitalize the surplus to the extent prevailing in the last few years." In the vast majority of cases the capitalization of surplus through stock dividends is a matter which is of no consequence either to the public or to the stockholders. The report embodies the results of an investigation well made, but not worth making.

CHARLES O. HARDY

Institute of Economics

Petroleum and Coal, the Keys to the Future, by W. T. Thom, Jr. Princeton: Princeton University Press. 1929. 223 pp.

At a time when both of the major fuel industries of the world are suffering from a tendency toward overproduction, it is especially helpful to have this review of the world's reserves of coal and oil from the pen of a geologist of distinction. *Petroleum and Coal, the Keys to the Future* is quite the best critical summary of our present knowledge of the original occurrence, geological distribution, and aggregate reserves of the mineral fuels. The book deserves intensive study by those interested in geography and industry and should be widely used by workers in the social sciences who desire an authoritative statement of present geologic opinion. Chapters sketching the importance of fuel to present and future civilization and outlining some of the outstanding economic problems serve as a background to the discussion of reserves which is the primary purpose of the book.

The author is peculiarly fitted for his task. At present professor of geology at Princeton, he was formerly chief of the section of oil and coal geology of the United States Geological Survey. Long experience in the field and laboratory investigations of the Survey, participation in its economic studies, and observations in foreign lands have rounded out his exceptional equipment.

In discussing the coal fields of the world, the author stresses the primary importance of coking coal in determining the location of the iron and steel industry. Recent advances in the fuel efficiency of the blast furnace, reducing the consumption of coke per ton of iron ore, have not altered the verdict of experience that the ore tends to move to the fuel rather than the fuel to the ore. In addition to the requirements of coke manufacture, the consumption in steel works, rolling mills, and the associated heavy industries raises the ratio of coal to ore to more than two to one.

In connection with the distribution of petroleum reserves, the author emphasizes the danger of international friction unless nations adopt some equitable plan for the free movement of the product in international trade. Recent geologic study serves but to emphasize the unequal distribution of mineral fuels. Africa and Australia lack major reserves of either oil or coal. South America is rich in oil but poor in coal, and Europe is rich in coal but, outside of the Black Sea region, poor in oil. Asia, in different parts of its vast expanse, is rich in both coal and oil. North America contains more than half the coal and, with readily accessible supplies from the Caribbean region, is heavily endowed with oil. The greatest store-house of oil is probably the region of Tertiary sediments extending from the Black Sea and the Caspian to the Persian Gulf.

A concluding chapter treats of the aggregate reserves of the world in relation to the possible demands. The bearing of future developments in the technique of exploration and production is illustrated by the outlook for petroleum. Past estimates of American reserves, such as those of 1922 which placed the oil *recoverable by methods then in use* at nine billion barrels, can now be greatly increased, not so much because of discovery of new deposits as because of engineering progress in drilling to deeper sands and in recovering a larger percentage of the oil imprisoned in the sand. Since 1922 the practicable depth of drilling has increased from 5,000 to 8,500 feet, and the proportion of oil in the sand which can be recovered through secondary operations has increased from perhaps 15 to nearer 50 per cent.

The outlook for the world's supplies in the future is summarized as follows:

> Our coal reserves appear to be large enough to last, with due economy of use, for a thousand years or more—that is to say, far beyond the range of valid prediction. The world's reserves of oil practically available by methods now in use, although of scant size when viewed from a broad national or humanistic standpoint, appear to be so capable of multiplication (in so far as practical effect is concerned) by economies of use, by improvements of discovery and production, and by the use of supplementary sources of energy, that the situation is one calling for sound judgment and careful planning for the future rather than for pessimism and alarm. Increased recoveries of gasoline from natural gas and of motor fuel from coke-oven operations, the artificial conversion of coal into liquid fuel products and the possible conversion of natural gases into gasoline, together with the revolutionary reduction in the quantities of coal required for major power purposes, and equally great prospective economies in the use of oil products, all go to assure us that if the public actively interests itself in the support of science, and if scientists really devote themselves to the pursuit of knowledge for humanity's sake, we will never be destitute of the necessities of life, although we likewise may never escape from the necessity of earning our living.

The author's treatment would be strengthened by a more extended reference to the tendency toward increasing cost, arising through the increasing difficulties of mining. Absolute exhaustion of any mineral is a distant and remote thing; increasing cost due to exhaustion of the richest deposits is often imminent. The state of Maryland is credited by Professor Thom with 8,043,000,000 tons of coal originally in the ground. The quantity mined to date is 220,000,000 tons, and even allowing for waste in mining, at least 94 per cent of the original tonnage is still there; yet exhaustion of the thickest and most accessible coal has so increased the difficulty of mining that the output of man per day has fallen from 4.05 tons in the late nineties to 3.47 tons in the period since the war. Similarly, the coal produced in England to date is barely 8 per cent of the original tonnage according to the very careful and probably correct estimates of the geologists. Yet the output per worker has been declining since 1883, and with 92 per cent of the original reserve still intact, the British are driven to use seams as thin as 18 inches and to work at depths as great as 3,500 feet. The technical progress which in other industries results in a rapid and continuous increase in output per man tends in mining to be absorbed by the increasing difficulties of nature.

Professor Thom's style is vigorous, clear, and pleasing. On March 21, 1929, the Geological Survey celebrated its fiftieth anniversary, and not the least of its services has been the standard of clear and simple exposition of technical matter impressed upon its graduates.

F. G. TRYON

Washington, D. C.

RECENT LITERATURE

DUE to the appearance in March of the new journal

SOCIAL SCIENCE ABSTRACTS

which is designed to cover fully and for the several social sciences the field dealt with superficially in the Recent Literature section of our JOURNAL, this section will henceforth be discontinued.

Social Science Abstracts will be published monthly and will afford thorough abstracts of new material appearing in periodicals, government publications, bulletins, reviews, etc. It will cover foreign as well as local sources in the fields of cultural anthropology, economics, history, human geography, political science, sociology and statistics.

For further information the reader is referred to this JOURNAL, June, 1928, Vol. 23, No. 162, p. 187, and December, 1928, Vol. 23, No. 164, p. 448, and to

THE EDITOR, *Social Science Abstracts*,
611 Fayerweather Hall, Columbia University,
New York City.